The Role of Upstream-Downstream Competition on Bundling Decisions: Should Regulators Force Firms to Unbundle?

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Abstract

We develop an upstream-downstream model to analyze downstream firms’ incentives to bundle. In our framework, the upstream firms are content providers (such as television stations) and the downstream firms are network operators (such as cable/satellite providers). We show that an a la carte regulation (i.e., a regulation that forces downstream firms to unbundle) leads to higher consumer surplus, if the unregulated equilibrium exhibits pure bundling. Hence, our model predicts that in the television industry, which is mainly characterized by pure bundling, an a la carte regulation will be beneficial for the consumers. If, on the other hand, the unregulated equilibrium is characterized by mixed bundling, then an a la carte regulation will increase consumer welfare if and only if the share of the bundled purchases for any downstream firm is large enough (relative to the shares of the single purchases).

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“Letters have been streaming into my office. I don’t hear as much about highlighted issues, like gay marriage... as I do about rising cable rates. The logical next step is to relieve consumers of the burden of paying for channels they don’t watch.” – Senator Frank Lautenberg (Pittsburgh Tribune Review, “A la Carte Cable Served Up as Solution,” March 27, 2004)

1 Introduction

The purpose of this paper is to examine firms’ incentives to bundle and the welfare implications of bundling in an upstream-downstream model. More specifically, there are two upstream firms, each producing one intermediate good and two downstream firms. Each downstream firm can potentially use both intermediate goods in producing its final product, which is then sold to consumers. The model we develop is particularly useful to study competition in a market where the upstream firms are content providers and the downstream firms are network operators, which offer various contents via their networks to consumers (e.g. wireless industry, Internet Service Providers, cable television industry, satellite television, etc). In such a content-network context, a downstream network operator may offer each content separately, or the two contents together as a bundle (pure bundling), or both (mixed bundling). In this paper, we focus primarily on the scenario where upstream content providers are television stations (such as ESPN, CNN, etc.) and the downstream networks are cable or satellite television providers (such as Comcast or DirecTV), but our set-up can also be thought of in a more general content-network setting.

There is considerable public interest in regulating the television market. For example, a number of consumer advocacy groups and politicians have expressed dissatisfaction with the current state of cable television pricing. An October 2003 study by the U.S. General Accounting Office (GAO) found that cable television rates had increased approximately 40 percent in the preceding five years. This is in contrast to the 12 percent increase in general inflation during the same time period. Consumers and politicians, alike, have been seeking out possible remedies for these rising rates. One of the most oft-cited potential solutions is to allow consumers to purchase cable stations “a la carte.” Under the current system, consumers typically have a choice between two or three different cable packages. Each of these “packages” is essentially a bundle of a number of different

1 “We are confident that once the FCC looks past industry innuendo and unsubstantiated assertions to find the real facts about the benefits of allowing consumers to select and pay for their own channels, cable and satellite carriers will no longer be able to resist public pressure to offer such options.” – Consumers Union (“FCC Asked to Examine A la Carte Cable TV,” Washington Post, May 20, 2004). See, also, “Cable TV Could Get Its Mouth Washed Out,” Business Week, May 2, 2005.
cable stations.\textsuperscript{2} As Senator McCain’s quotation in the footnote below indicates, under such a system, consumers are forced to purchase products for which they may have no particular interest. For example, a sports fan that purchases Comcast’s basic cable package to receive ESPN is also “purchasing” stations such as PBS, ABC Family, and Nickelodeon. A puzzle, indeed, is why the selection choice offered to viewers is so limited. Proponents of a la carte pricing argue that allowing consumers to purchase stations individually will give them greater control over their cable bill and, presumably, lower the cost of cable for the average viewer. The above conclusion relies upon a rather strong assumption, however: that cable companies would not respond to a la carte regulations by changing the prices charged for each station. If cable providers respond to a la carte regulations, by raising prices, then it is not clear that a la carte pricing will be welfare-improving. The model we develop can be used to offer some guidance as to whether downstream firms should be forced to unbundle.

We assume that the two contents and the two networks—as stand alone goods—are horizontally differentiated. Consumers must subscribe to one and only one network, but are allowed to purchase one or both contents. In our framework, each consumer has a most preferred content, but also appreciates content variety. In particular, consumers derive an incremental utility from consuming a second content. If the incremental utility parameter is high relative to the cost (price) of purchasing a second content, then some consumers find it profitable to make multiple content purchases.\textsuperscript{3}

In sections 2 and 3 we present an analysis where we assume that bundling is allowed. We find (proposition 1) that if the incremental utility from consuming a second content is low, then the equilibrium is characterized by \textit{no bundling}. If the incremental utility is in an intermediate range, then the equilibrium is characterized by \textit{mixed bundling}. Finally, if the incremental utility is sufficiently high, then firms offer only the bundle (i.e., \textit{pure bundling}). In section 5 we assume that a regulator forces downstream firms to unbundle (an a la carte regulation). Even in this case, consumers can create the bundle, themselves, by purchasing both contents. The difference with the previous case, where bundling is allowed, is that now consumers who buy a second content do not receive a discount. If the incremental utility parameter exceeds a threshold, then some consumers find it worthwhile to buy both contents (proposition 2). We, then, compare the welfare properties

\textsuperscript{2} “When I go to the grocery store to buy a quart of milk, I do not have to buy a package of celery and a bunch of broccoli. I do not like broccoli.” Senator John McCain (March 26, 2004).

\textsuperscript{3} Crampes and Hollander (2005) make a similar assumption. Kim and Serfes (2004) allow for multiple purchases in a Hotelling-type model and analyze the firms’ location decisions.
of the two regimes (subsection 5.3).

We find that if, in the unregulated equilibrium, downstream firms offer only the bundle (pure bundling), then an a la carte regulation will increase consumer surplus. Hence, our model predicts that in the cable television industry, which is mainly characterized by pure bundling, an a la carte regulation will be beneficial for the consumers.\(^4\) Recently, the new commissioner of the FCC disavowed an earlier FCC study and advocated an a la carte pricing for the TV industry (“FCC Reverses Course on Cable, Backing A-La-Carte Pricing,” Wall Street Journal, November 29, 2005). Our model provides theoretical support for this decision. If the unregulated equilibrium exhibits mixed bundling, then an a la carte regulation will increase consumer welfare if and only if the share of the bundled purchases for any downstream firm is large enough (relative to the shares of the single purchases).

The intuition behind our main results is as follows. We identify three effects that are associated with an a la carte regulation: i) an \textit{upstream effect}, ii) a \textit{surplus extraction effect} and iii) a \textit{change in the intensity of competition effect}. First, let’s explain how each effect works.

We begin with the first effect. The “upstream” effect focuses on the upstream firms’ incentives to undercut each other’s price. We argue that upstream firms compete more vigorously when bundling is \textit{not} permitted. Because the upstream firms’ prices may be thought of as the downstream firms’ marginal costs (and because consumers receive some of the benefit of lower costs via lower prices), we find that the “upstream” effect of a la carte regulation is beneficial to consumers. To see why this is the case, consider a market with two upstream rivals (CNN and ESPN) and two downstream rivals (Comcast and DirecTV) each offering the two contents separately and together (at a discount) as a bundle (mixed bundling). When bundling is allowed, CNN (for example) has less incentive to lower their price. Why? By (unilaterally) lowering their price, CNN is creating a positive externality for ESPN. This is because the resulting lower price of the bundle increases purchases of ESPN (through the bundle). In this manner, ESPN may wish to keep their own price high, while CNN lowers theirs. When bundling is prohibited, this positive externality no longer exists. Each firm receives the full benefit of any price reduction (along with the network providers). This makes undercutting more rewarding for the upstream firms. So, our model predicts that an a la carte regulation will lower the upstream prices and part of this decrease will be passed on to consumers.

\(^4\)The fact that the TV industry is predominantly characterized by pure bundling is also supported by empirical evidence. Crawford (2005) finds that mixed bundling is not very common in the TV industry.
Now we elaborate on the remaining two effects, starting with the “surplus extraction” effect. The bundle helps the downstream firms to reduce consumer heterogeneity and extract more surplus (e.g. second-degree price discrimination). An a la carte regulation will eliminate the surplus extraction effect and raise consumer surplus. Finally, the “change in the intensity of competition” effect works as follows. The ability of downstream firms to bundle affects the number of products that each firm can offer. If that number increases, then firms compete on more fronts and competition intensifies. An important question then arises: when should we expect the number of products to increase following an a la carte regulation? If the unregulated equilibrium is characterized by mixed bundling, then an a la carte regulation will reduce the number of available products (because the bundle is eliminated) and downstream competition will diminish, leading to a lower consumer surplus. If, on the other hand, the unregulated equilibrium is characterized by pure bundling, then an a la carte regulation will increase the number of products offered by downstream firms and competition will intensify (because before the regulation each downstream firm offers only one product, the bundle).

We have seen so far that the “upstream” and the “surplus extraction” effects will impact consumer surplus positively if an a la carte regulation is passed, while the impact of the “change in the intensity of competition” effect on consumer welfare is ambiguous and depends critically on whether the unregulated equilibrium is characterized by mixed or pure bundling. If the unregulated equilibrium exhibits pure bundling then the “change in the intensity of competition” effect also affects consumer welfare positively. Its impact, however, is negative when the unregulated equilibrium exhibits mixed bundling.

Therefore, the next important question is: when should we expect firms to “play” a mixed bundling equilibrium and when not? To induce positive sales for all of its products, a downstream firm must offer the bundle at a discount. When the consumer preference for content variety is strong firms do not have adequate incentives to offer the bundle and the contents separately. This would require them to offer the bundle at a discount, but precisely because demand for both contents is strong this strategy is not profitable. Firms are better off eliminating the discount and offering only the bundle. In this case the unregulated equilibrium is characterized by pure bundling. On the other hand, if the consumer preference for content variety is not strong, then mixed bundling emerges in equilibrium and if that preference is low then firms do not offer the bundle at all (no bundling). This latter case is also of interest. Why would the downstream firms not offer the
bundle? The reason is that, when demand for variety is weak, the upstream firms charge very high prices. The marginal costs of the downstream firms are high and hence have no incentives to bundle the contents and offer them at a discount.

To summarize, if in the unregulated equilibrium downstream firms offer only the bundle (pure bundling), then an a la carte regulation will increase the consumer surplus. All three effects point in the same direction. If the unregulated equilibrium is characterized by mixed bundling, then an a la carte regulation will increase consumer surplus if and only if the “upstream” and “surplus extraction” effects dominate the “change in the intensity of competition” effect. We show that this is the case if the consumer preference for content variety is strong. This is equivalent to saying that the share of bundled purchases, for any downstream firm, is large enough (relative to the shares of the single purchases).

Furthermore, firms do not become worse off when they are able to offer the bundle. On the contrary, both the upstream and the downstream firms are better off with the bundle. This may explain why the cable industry is opposed to legislation that would have required the companies to offer a la carte pricing.\(^5\) Crawford (2005), using TV Cable data, finds strong support for the discriminatory theory of bundling. Bundling an average top-15 cable channels is estimated to increase profits and reduce consumer welfare, with an average effect of 6% and 5.5% respectively. These predictions are qualitatively consistent with our theoretical results.

There is a large theoretical literature on bundling. In a monopoly setting, mixed bundling will increase the monopolist’s profits [e.g. Adams and Yellen (1976) and McAfee et al. (1989)]. Fang and Norman (2005) examine a multi-product monopolist’s incentives to bundle. They compare pure bundling with no bundling. In an oligopolistic market, bundling may lead to a prisoners’ dilemma [e.g. Economides (1993) and Reisinger (2004)]. Matutes and Regibeau (1992) investigate the issue of bundling in a duopoly model with complementary goods (software-hardware). They also show that for a range of parameters the game is a prisoners’ dilemma. Anderson and Leruth (1993) show, in a duopoly model, that mixed bundling leads to lower profits.\(^6\) Bundling may also

\(^5\)When articles in the business press on cable pricing refer to an “a la carte regulation” they do not make it clear whether it is: i) a regulation that prohibits downstream firms from offering the bundle, or ii) a regulation which forces downstream firms to offer the products separately. Under the “latter type” of regulation, firms can bundle the two contents if they want to. Throughout the present paper, an “a la carte regulation” is meant to imply the “former type” of regulation. Our analysis highlights the difference between these two types of regulation and shows that the “former type” is more effective (see section 7).

\(^6\)In section 6, we explain why our model does not lead to a prisoners’ dilemma.
be used by an incumbent firm as an entry foreclosure device [e.g. Whinston (1990) and Nalebuff (2004)]. O’Brien and Shaffer (2005) examine the effects of horizontal mergers in intermediate goods markets, where upstream firms may have the ability to bundle their products. Stole (2003) offers a more systematic review of the bundling literature.

Nevertheless, there is not much work done on bundling (and unbundling) final goods in an upstream-downstream framework. Such an issue may not be particularly interesting in more traditional industries, but we believe that it is becoming increasingly important especially in our digital era where various kinds of contents (e.g. news, movies, games, etc.) are reaching consumers via different types of networks (e.g. Internet, mobile communications, satellite television, etc.). Pricing decisions made at the upstream level determine, to a large extent, the downstream firms’ decisions to bundle, as well as the effect of unbundling on welfare (e.g. the “upstream” effect). Our model is rich enough in that it yields different market configurations (e.g. no, mixed, or pure bundling) as equilibrium outcomes. Identifying the conditions under which each kind of an equilibrium will emerge is important, as each type of an outcome may be applicable to different industries. The main innovations of this paper are: i) the explicit modeling of the vertical channel, ii) the introduction of competition both at the upstream and downstream levels and iii) the examination of the effectiveness of a regulation that forces downstream firms to unbundle.

The remainder of the paper is organized as follows. The main model is presented in section 2 and the equilibrium analysis, where bundling is allowed, is presented in section 3. In section 4, we derive the first best and the welfare under the non-cooperative outcome of section 3. In section 5, we derive the equilibrium assuming that downstream firms are not allowed to bundle and we compare the welfare properties of this equilibrium with the one we derived in section 3. We offer a discussion about the robustness of our predictions in section 6. We conclude in section 7. The proof of proposition 1 can be found in the appendix.

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7 Bakos and Brynjolfsson (2000) model the decision to bundle in upstream and downstream competition for information goods, but their model is significantly different from ours. In their model the upstream competition is between firms for the exclusive rights of an information good and downstream competition is between firms who compete for consumers. Moreover, each stage is analyzed separately and there is no link between the upstream and the downstream stages.
2 Description of the Model

The market consists of two upstream firms (content providers), denoted by $U_j$ ($j = a, b$) and two downstream firms (network operators), denoted by $D_i$ ($i = 1, 2$). Each downstream firm will potentially use both contents as inputs and consumers can subscribe to only one network. This implies that there are, in total, four products, $1a, 1b, 2a$ and $2b$ (we have not introduced bundled products yet). For example, $1a$ is content $a$ offered by network 1. We assume that contents and networks, as stand alone products, are horizontally differentiated. More specifically, we assume that the population of consumers is divided into two groups: those who prefer network operator 1 and those who prefer network operator 2, as stand alone goods, all else equal.\(^8\) Within each group there are two subgroups of consumers: those who prefer content $a$ and those who prefer content $b$, all else equal.\(^9\) We capture this kind of differentiation, by placing the four products equidistantly on the Salop circle [Salop (1979)], of unit circumference (see figure 1). The consumers who are located on the northwest part of the circle are the ones with stronger preference for network 1 and those located on the southeast part of the circle prefer network 2. Consumers are uniformly distributed on the circle and have measure one.

We assume that downstream firm $i$ can offer three products: $a$ and $b$ separately, denoted by $ia$ and $ib$ respectively and $a$ and $b$ together as a bundle, denoted by $iB$. We denote by $p_{ia}$, $p_{ib}$ and $p_{iB}$

\(^8\)This could be due, perhaps, to (perceived) differences in customer service or other features.

\(^9\)Say, sports channels versus news channels.
the prices of the available products. Moreover, we denote by \( ij \) the location of content \( j \) offered by downstream firm \( i \) and by \( iB \) the bundle offered by downstream firm \( i \). Therefore, there exists a maximum of six products in the market: the four described in the previous paragraph and the two bundles. Consumers purchase only one product and incur a linear disutility from not being able to buy their “ideal” product. Each consumer has a most preferred content, but also appreciates content variety.\(^{10}\) We capture this feature as follows. Consider a consumer who is located at \( x \). Her indirect utility is given by,

\[
U = \begin{cases} 
V(a) - t|x - ia| - p_{ia}, & \text{if she buys product } ia \\
V(b) - t|x - ib| - p_{ib}, & \text{if she buys product } ib \\
V(a, b) - t|x - ia| - t|x - ib| - p_{iB}, & \text{if she buys the bundle } iB.
\end{cases}
\]

We assume that \( V(a, b) \geq V(a) = V(b) = V \) and we set \( V(a, b) = V + \theta \), where \( 0 \leq \theta \leq V \). Furthermore, \( V \) is assumed to be sufficiently high so that the market is covered. The parameter \( \theta \) measures the incremental utility from consuming a second content. By limiting \( \theta \), we are assuming that there is a benefit from receiving two stations (holding price and distance constant), but that consumers experience diminishing marginal utility from consuming the second content [see also Kim and Serfes (2004)].\(^{11}\) Consumers choose the product that maximizes their indirect utility. The content prices that the upstream firms charge (in the form of take-it-or-leave-it offers that are also equal across downstream firms) are denoted by \( r_a \) and \( r_b \) respectively. These prices enter into the marginal costs of the downstream firms.\(^{12}\)

We will analyze the following two-stage game, with simultaneous moves in each stage.

- **Stage 1** (*Input pricing decisions*): The upstream firms choose the prices \( r_a \) and \( r_b \).
- **Stage 2** (*Product pricing decisions*): The downstream firms choose the prices of their products, \( p_{ia} \), \( p_{ib} \) and \( p_{iB} \).

In solving for an equilibrium, in section 3, we assume that bundling is allowed. In section 5, we

\(^{10}\) This feature allows us to avoid some of the problems that are present in standard location models, which ignore any aggregate demand creation and focus entirely on business stealing effects.

\(^{11}\) For tractability, we assume that the incremental utility parameter \( \theta \) is the same across all consumers.

\(^{12}\) Our model ignores the possibility that an a la carte regulation is likely to increase the costs that are associated with “managing” all the separate channels (rather than the bundle). These costs could be administrative, billing or marketing costs. Nevertheless, these additional expenses (after an a la carte regulation) are likely to be fixed rather than variable. Therefore, they will have no significant impact on prices and on consumer welfare. Furthermore, advertising revenue are also assumed away. For a discussion on this we refer the reader to section 6.
solve for an equilibrium under the assumption that a regulator prohibits downstream firms from bundling the two goods, in order to assess the welfare implications of bundling.

There are possibly three different types of (symmetric) equilibria: i) no bundling equilibrium, where each downstream firm offers only the two products separately, ii) mixed bundling equilibrium, where each downstream firm offers the two goods separately and together as a bundle and iii) pure bundling equilibrium, where each downstream firm offers only the bundle. We search for a subgame perfect Nash equilibrium (SPNE) in pure strategies.

**Remark:** Due to the unit-dimensional characteristics space we have assumed, competition is localized (as in standard location models). For example, product 1a does not compete directly with product 2b. Also, as figure 2 demonstrates, the two bundles do not compete head-on with each other (in the mixed bundling equilibrium; in the pure bundling equilibrium they do). This feature of our model is likely to impact the “change in the intensity of competition” effect. A less restrictive modeling framework would be a two-dimensional characteristics space [e.g. Matutes and Regibeau (1992)]. In our context one dimension would capture consumer preference heterogeneity for the contents and the other for the networks. That framework, however, would increase the complexity of the analysis in our set-up dramatically, because, among other things, we explicitly model the vertical channel and competition both at the upstream and downstream levels. Hence, one can view the one dimensional characteristics space (circle) that we adopt in this paper as a way to obtain clean results and predictions, while still maintaining many of the salient features of differentiation at the upstream and downstream levels. Moreover, we believe that the effects we have identified are robust and our main results will hold qualitatively in a more general model of “global” competition. In section 6, we offer a discussion on the robustness of our results.

### 3 The Model & Equilibrium Analysis

#### 3.1 Consumer Demand

As shown in figure 2, we assume that 1a is located at 0, 1b and 2a at $\frac{1}{4}$ and 2b at $\frac{1}{2}$. Figure 2 depicts the mixed bundling configuration. There are six indifferent consumers on the circle: 1) consumer $x_1$ who is indifferent between 1b and 1B, 2) consumer $x_2$ who is indifferent between 1a and 1B, 3) consumer $x_3$ who is indifferent between 1b and 2b, 4) consumer $x_4$ who is indifferent between 1a and 2a, 5) consumer $x_5$ who is indifferent between 2a and 2B and 6) consumer $x_6$ who
is indifferent between $2b$ and $2B$ (see figure 2).

The equations of the locations of the marginal consumers are given in appendix I. Using these equations, the demand functions are given by,

\begin{align*}
  d_{1a} &= x_4 + x_2 = \frac{-8\theta - 12p_{1a} + 4p_{2a} + 8p_{1B} + 3t}{8t} \\
  d_{1b} &= x_3 - x_1 = \frac{-8\theta - 12p_{1b} + 4p_{2b} + 8p_{1B} + 3t}{8t} \\
  d_{1B} &= x_1 - x_2 = \frac{8\theta - 8p_{1B} + 4p_{1a} + 4p_{1b} - t}{4t} \\
  d_{2a} &= x_5 - x_4 = \frac{-8\theta - 12p_{2a} + 4p_{1a} + 8p_{2B} + 3t}{8t} \\
  d_{2b} &= \left(\frac{1}{2} - x_3\right) + \left(\frac{1}{2} - x_6\right) = \frac{-8\theta - 12p_{2b} + 4p_{1b} + 8p_{2B} + 3t}{8t} \\
  d_{2B} &= x_6 - x_5 = \frac{8\theta - 8p_{2B} + 4p_{2a} + 4p_{2b} - t}{4t}.
\end{align*}

### 3.2 Equilibrium - Bundling is Allowed

The next proposition summarizes the SPNE of this game.

**Proposition 1 (Bundling is allowed).** The SPNE is described as follows,
NO BUNDLING (NB). (Low preference for content variety). If $0 \leq \theta \leq \left(\frac{4\sqrt{66} - 11}{24}\right)t \approx .896t$, then firms have no incentive to offer the bundle. Upstream prices and profits are given by,

\[ r_a = r_b = \frac{5t}{6} \text{ and } \Pi_{U_a} = \Pi_{U_b} = \frac{5t}{12}. \]

Downstream prices and profits are given by;

\[ p_1 = p_2 = p_{1b} = p_{2b} = \frac{4t}{3} \text{ and } \Pi_{D_1} = \Pi_{D_2} = \frac{t}{4}. \]

MIXED BUNDLING (MB). (Medium preference for content variety). If $\left(\frac{4\sqrt{66} - 11}{24}\right)t \approx .896t < \theta < \frac{135\theta}{88} \approx 1.534t$, then firms offer the bundle and the individual goods separately. Upstream prices and profits are given by,

\[ r_a = r_b = \frac{10\theta}{21} + \frac{5t}{28} \text{ and } \Pi_{U_a} = \Pi_{U_b} = \frac{55(3\theta + 8\theta)^2}{14112t}. \]

Downstream prices and profits are given by;

\[ p_1 = p_2 = p_{1b} = p_{2b} = \frac{10\theta}{21} + \frac{19t}{28}, p_{1B} = p_{2B} = \frac{17\theta}{14} + \frac{79t}{112} \text{ and } \]
\[ \Pi_{D_1} = \Pi_{D_2} = \frac{7744\theta^2 + 16713\theta^2 - 8976\theta t}{56448t}. \]

PURE BUNDLING (PB). (High preference for content variety). If $\theta \geq \frac{135\theta}{88} \approx 1.534t$, then firms offer only the bundle. Upstream prices and profits are given by,

\[ r_a = r_b = \theta - \frac{5t}{8} \text{ and } \Pi_{U_a} = \Pi_{U_b} = \theta - \frac{5t}{8}. \]

Downstream prices and profits are given by;

\[ p_{1B} = p_{2B} = 2\theta - \frac{t}{2} \text{ and } \Pi_{D_1} = \Pi_{D_2} = \frac{\theta}{2} + \frac{t}{16}. \]

The proof can be found in appendix II.
3.3 Discussion of the Equilibrium

There are three types of equilibria in this model. If the incremental utility $\theta$ from consuming a second content ($a$ or $b$) is low (i.e., $0 \leq \theta \leq \left(\frac{4\sqrt{66}-11}{24}\right)t \approx .896t$), then the upstream firms (content providers) find it optimal to charge relatively high prices to the downstream firms (network operators), see figure 3. This forces the downstream firms to charge high prices for their products. This response is optimal precisely because consumers do not value a second content much and firms focus on single content purchases. As a consequence, the price of the bundle would be high (relative to the incremental benefit $\theta$) and no consumer would purchase the bundle. This implies that downstream firms do not offer the bundle (NB). If the incremental utility is in a medium range (i.e., $\left(\frac{4\sqrt{66}-11}{24}\right)t \approx .896t < \frac{135t}{88} \approx 1.534t$), then the upstream firms lower discretely their prices (we explain why the jump at $\theta = \left(\frac{4\sqrt{66}-11}{24}\right)t$ in figure 3 occurs in the next paragraph). However, as $\theta$ increases price rises as figure 3 demonstrates. The downstream firms now offer the bundle and each content separately, mixed bundling (MB) (see figure 4). Finally, if the incremental utility is high (i.e., $\theta \geq \frac{135t}{88} \approx 1.534t$), then only the bundle is offered (PB). The upstream firms charge high prices and the downstream firms cannot offer each good separately at a price that would generate positive demand.

The profit functions of the upstream firms are not quasi-concave (see figure 5 below where we plot the profit function of $U_a$). To draw figure 5 we have fixed $r_b = \frac{t}{2}$ and $\theta = .65t$. The profit function is the upper envelope of the two separate functions. The function that peaks first, depicts
the profits when the downstream firms offer the bundle (mixed bundling), whereas the function that peaks second, shows the profits under no bundling. Figure 5 helps us gain a better grasp of the problem. The upstream firms can pursue two distinct types of strategies: i) set a low price and induce the downstream firms to offer the bundle, or ii) charge a high price (upward jump) and eliminate the bundle. When \( \theta \) is high, the first strategy is more profitable, while for low \( \theta \) the second strategy yields higher profits. The threshold of \( \theta \), where this switching takes place, is \( \frac{t(4\sqrt{66}-11)}{24} \) (see figures 3 and 4). Pricing decisions made by the upstream firms affect downstream firms’ incentives to offer the bundle. For a range of the parameter \( \theta \) [see (43)] there are two equilibria: one with high upstream prices and no bundle at the downstream level and the other with low upstream prices and mixed bundling at the downstream level. Upstream firms, as we show in appendix I, prefer the first one.

Moreover, the lack of quasi-concavity of the upstream firms’ profit functions, does not lead to a non-existence of a pure strategy equilibrium. This is because the upstream firms’ best-response correspondences have only upward jumps (details are omitted). Hence, an equilibrium in pure strategies must exist [see Vives (1999, Theorem 2.5, p.33)].

Figure 5: Upstream profit function
4 Welfare analysis

4.1 First best

The social surplus, assuming that $x_2$ and $x_1 \in [0, \frac{1}{4}]$ and $x_5$ and $x_6 \in [\frac{1}{4}, \frac{1}{2}]$ (i.e., as shown in figure 2), is the difference between aggregate benefit and transportation cost and it is given by,

$$SS = \int_0^{x_2} [V(a) - tx] dx + \int_{x_2}^{x_1} [V(a, b) - \frac{t}{4}] dx + \int_{x_1}^{x_5} [V(b) - t \left(\frac{1}{4} - x\right)] dx +$$

$$+ \int_{x_5}^{x_6} [V(a, b) - t \left(\frac{1}{2} - x\right)] dx \quad \text{and} \quad \text{if and only if} \quad x_1 = \frac{\theta}{t}, \quad x_2 = \frac{-4\theta + t}{4t}, \quad x_3 = \frac{3}{8}, \quad x_4 = \frac{1}{8}, \quad x_5 = \frac{t - 2\theta}{2t} \quad \text{and} \quad x_6 = \frac{t + 4\theta}{4t}. \quad (8)$$

In the equally plausible case where $x_2 \leq 0$ and $x_6 \geq \frac{1}{2}$ (consult again figure 2, but now $x_2$ is located on the right half of the circle and $x_6$ on the left half), then the social surplus is the same as in (7), except that $x_2$ becomes $-x_2$ and $x_6$ becomes $1 - x_6$.

The first best locations,\(^{13}\) should maximize (7) and are given by,

$$x_1 = \frac{\theta}{t}, \quad x_2 = \frac{-4\theta + t}{4t}, \quad x_3 = \frac{3}{8}, \quad x_4 = \frac{1}{8}, \quad x_5 = \frac{t - 2\theta}{2t} \quad \text{and} \quad x_6 = \frac{t + 4\theta}{4t}. \quad (8)$$

As above, when $x_2 \leq 0$ and $x_6 \geq \frac{1}{2}$, the optimal locations are obtained from (8) through the following transformations: $x_2$ becomes $-x_2$ and $x_6$ becomes $1 - x_6$. The size of the group of consumers who buys the bundle is,

$$x_1 - x_2 = \frac{8\theta - t}{4t} \quad \text{and} \quad x_6 - x_5 = \frac{8\theta - t}{4t}.$$
4.2 Welfare Under the Non-Cooperative Outcome of Proposition 1

Now we compute the social welfare and the consumer surplus that the non-cooperative equilibrium (see proposition 1) generates. The locations of the marginal consumers, in the mixed bundling equilibrium (MB), are given by [after we substitute (35) into (31)],

\[
\begin{align*}
x_1 &= \frac{-1035t + 10120\theta}{38640t}, \\
x_2 &= \frac{10695t - 10120\theta}{38640t}, \\
x_3 &= \frac{3}{8}, \\
x_4 &= \frac{1}{8}, \\
x_5 &= \frac{20355t - 10120\theta}{38640t} \text{ and } x_6 = \frac{8625t + 10120\theta}{38640t}.
\end{align*}
\]

(9)

In the pure bundling equilibrium (PB), the locations of the marginal consumers are,

\[
x_2 = x_4 = x_5 = \frac{1}{8} \text{ and } x_1 = x_3 = x_6 = \frac{3}{8}.
\]

(10)

The locations of the marginal consumers, in the no bundling equilibrium (NB), are given by,

\[
x_1 = x_2 = \frac{1}{8}, \quad x_3 = \frac{3}{8}, \quad x_4 = \frac{1}{8} \text{ and } x_5 = x_6 = \frac{3}{8}.
\]

(11)

The social surplus under the mixed bundling (MB) non-cooperative equilibrium, after we substitute (9) into (7), is given by,

\[
SS_{MB} = \frac{56448Vt - 32688\theta t + 51392\theta^2 - 1845t^2}{56448t}.
\]

(12)

The social surplus in the pure bundling (PB) non-cooperative equilibrium, after we substitute (10) into (7), is given by,

\[
SS_{PB} = V + \theta - \frac{5t}{16}.
\]

(13)

Under the no bundling (NB) non-cooperative equilibrium, after we substitute (11) into (7) the social surplus is,

\[
SS_{NB} = V - \frac{t}{16}.
\]

(14)

Figure (6) depicts the social surplus as a function of the incremental utility parameter \(\theta\).
The upward jump in the social welfare occurs because when $\theta$ exceeds $\left(\frac{4\sqrt{66} - 11}{24}\right)t \approx .896t$, the upstream firms find it profitable to switch to the second equilibrium (recall, from the discussion in section 3.3, that for a range of the $\theta$ parameter there are two equilibria which can be Pareto ranked from the upstream firms’ perspective) by discretely lowering their prices. The upstream price decrease induces the downstream firms to offer the bundle, which in turn increases “total output” and improves efficiency (and consumer surplus—see the first upward jump in figure 7).

Consumer surplus is the difference between social surplus and aggregate profits and it is given, in the mixed bundling equilibrium, by,

$$CS^{MB} = \frac{56448Vt - 35856\theta t + 7744\theta^2 - 39231t^2}{56448t}.$$  \hfill (15)

In the pure bundling equilibrium it is given by,

$$CS^{PB} = V - 2\theta + \frac{13t}{16}.$$  \hfill (16)

In the no bundling equilibrium, consumer surplus is given by,

$$CS^{NB} = V - \frac{67t}{48}.$$  \hfill (17)

Figure 7 depicts the consumer surplus. The second downward jump in figure 7 occurs because firms after that threshold do not offer the contents for sale individually. This forces the consumers who used to purchase only one content to purchase the bundle and pay a much higher price (see figure 4). The downward jump in figure 7 best captures the discriminatory effects of bundling.
5 A Regulator Forces Downstream Firms to Unbundle

In this section, we assume that the downstream firms are no longer allowed to bundle the two goods. This could be due, for example, to regulation that requires cable/satellite providers to offer a la carte cable pricing. A consumer, if she wishes, she can purchase both contents from a downstream firm (network operator). In other words, consumers can create the bundle themselves.\footnote{We, inherently, assume that a consumer creating the bundle does so using products purchased through a single downstream firm. Also, as in Nalebuff (2004), we assume that consumers can integrate the two contents as well as producers can.} We will solve for the equilibrium in this modified game and we will compare the welfare properties with those derived in the previous sections where bundling was allowed.

The marginal consumers are again given by (21-26), with the only difference that now \( p_{iB} = p_{ia} + p_{ib} \). \( B \) now stands for a bundle that is created by a consumer who purchases both products. Intuitively, some consumers will purchase both products of a downstream firm if the incremental utility \( \theta \) is high. This will make a second purchase worthwhile. The demand functions are given by (1-6), with \( p_{iB} = p_{ia} + p_{ib} \).

5.1 Equilibrium - Bundling is Not Allowed

The next proposition summarizes the SPNE when firms are not allowed to bundle.

**Proposition 2 (Bundling is not allowed).** The SPNE is described as follows,

**NO CONSUMER BUNDLES (NCB).** (Low preference for content variety). If \( 0 \leq \theta \leq \left( \frac{2\sqrt{20} - 3}{24} \right) t \approx 1.053t \), then in equilibrium no consumer makes multiple purchases. Upstream prices and profits are given by,

\[
r_a = r_b = \frac{5t}{6} \quad \text{and} \quad \Pi_{a} = \Pi_{b} = \frac{5t}{12}.\]

Downstream prices and profits are given by,

\[
p_{1a} = p_{2a} = p_{1b} = p_{2b} = \frac{4t}{3} \quad \text{and} \quad \Pi_{D_1} = \Pi_{D_2} = \frac{t}{4}.\]

**SOME CONSUMERS BUNDLE (SCB).** (Medium preference for content variety). If \( \left( \frac{2\sqrt{20} - 3}{24} \right) t \approx 1.053t < \theta < \frac{37t}{24} \approx 1.541t \), then some consumers make multiple purchases. Upstream prices and...
profits are given by,

\[ r_a = r_b = \frac{\theta}{2} + \frac{t}{16} \quad \text{and} \quad \Pi_{U_a} = \Pi_{U_b} = \frac{3(8\theta + t)^2}{640t}. \]

Downstream prices and profits are given by;

\[ p_{1a} = p_{2a} = p_{1b} = p_{2b} = \frac{7\theta}{10} + \frac{7t}{80} \quad \text{and} \quad \Pi_{D_1} = \Pi_{D_2} = \frac{3(8\theta + t)^2}{1600t}. \]

ALL CONSUMERS BUNDLE (ACB). (High preference for content variety). If \( \theta \geq \frac{37t}{24} \approx 1.541t \), then all consumers buy both downstream products. Upstream prices and profits are given by,

\[ r_a = r_b = \frac{\theta}{2} - \frac{17t}{24} \quad \text{and} \quad \Pi_{U_a} = \Pi_{U_b} = \theta - \frac{17t}{24}. \]

Downstream prices and profits are given by;

\[ p_{1a} = p_{2a} = p_{1b} = p_{2b} = \frac{3t}{8} \quad \text{and} \quad \Pi_{D_1} = \Pi_{D_2} = \frac{t}{6}. \]

The proof of proposition 2 is similar to the proof of proposition 1 and it is omitted.

The upstream prices in proposition 1 are higher than the upstream prices in proposition 2 (except in the no bundling equilibrium where they are the same). This is due to the “upstream” effect which suggests that upstream firms are less willing to lower their prices when downstream firms are allowed to bundle.

5.2 Discussion of the Equilibrium

As in the case where bundling is allowed (proposition 1), there are three types of equilibria. The first one is characterized by no consumer consuming both downstream products (NCB). The incremental utility is relatively low (i.e., \( 0 \leq \theta \leq \left( \frac{2\sqrt{20} - 3}{24} \right) t \approx 1.053t \)) and firms find it profitable to set high prices so that no consumer finds it worthwhile to purchase a second content. Similar to the case where bundling is allowed, the profit functions of the upstream firms are not quasi-concave (similar to figure 5). The second type of equilibrium is when \( \left( \frac{2\sqrt{20} - 3}{24} \right) t \approx 1.053t < \theta < \frac{37t}{24} \approx 1.541t \). In this case, upstream firms lower their prices and some consumers make multiple content purchases.
However, the threshold above which this switching takes place is higher than the threshold above which firms switch to the mixed bundling equilibrium when they are allowed to bundle, i.e., \((2\sqrt{20}/24) t \approx 1.053t\) vs. \((4\sqrt{66}/24) t \approx 0.896t\) (see figures 8 and 9 where both thresholds are depicted). Firms have stronger incentives to induce some consumers to consume both products when they have the ability to offer the bundle. The bundle acts as a sorting device and, therefore, firms do not lose much revenue from the consumers with extreme preferences for either one of the two contents. The third type of equilibrium emerges when \(\theta \geq \frac{37}{24} t \approx 1.541t\), in which case all consumers consume both contents (ACB).

5.3 Welfare Comparison

Now we look at the welfare properties of the equilibrium, when bundling is prohibited, and compare them with those that emerge when bundling is allowed. When some consumers purchase both downstream products the locations of the marginal consumers—when bundling is not allowed—are given by,

\[
x_1 = \frac{24\theta - 7t}{80t}, \quad x_2 = \frac{3(-8\theta + 9t)}{80t}, \quad x_3 = \frac{3}{8}, \quad x_4 = \frac{1}{8}, \quad x_5 = \frac{47t - 24\theta}{80t} \quad \text{and} \quad x_6 = \frac{13t + 24\theta}{80t}.
\]

We substitute the above locations into (7) to derive the social surplus in the case where some consumers bundle (SCB). This yields,

\[
SS^{SCB} = \frac{3200Vt - 149t^2 + 3264\theta^2 - 2384\theta t}{3200t}.
\] (18)

The locations of the marginal consumers when all consumers bundle (ACB) are the same with (10). Consequently, the social surplus is the same with (13). Finally, when no consumer bundles (NCB) the social surplus is the same with (14).

Figure (8) compares the social surplus between the two regimes: when bundling is allowed [eqs (14), (12) and (13)—represented by the solid line] and when it is not [eqs (14), (18) and (13)—represented by the dotted line]. (The solid line in figure 8 is the same with that in figure 6). The social surplus under the former regime is always (i.e., for any \(\theta\)) higher than that under the latter regime. Total output, in both regimes, is less than the first-best output (see section 4.1, where the first best was derived), but the output when bundling is allowed is higher than when it is not allowed. The intuition is as follows. First, when the bundle is available firms offer it at a discount which induces more consumers to buy it relative to the case where consumers bundle themselves.
Second, firms switch to the equilibrium where consumers buy the bundle sooner (i.e., in terms of the threshold in the $\theta$ dimension) when they can offer the bundle than when they are not allowed to offer the bundle. Both of these effects increase total output and efficiency.

The consumer surplus (social surplus minus aggregate profits) when some consumers bundle (SCB) is given by,

$$CS^{SCB} = \frac{3200Vt - 191t^2 + 576\theta^2 - 3056\theta t}{3200t}. \tag{19}$$

When all consumers bundle (ACB), it is given by,

$$CS^{ACB} = V - \theta + \frac{37t}{48}. \tag{20}$$

Finally, when no consumer bundles (NCB) consumer surplus is the same with (17).

Figure (9) compares the consumer surplus between the two regimes: when bundling is allowed [eqs (17), (15) and (16)–represented by the solid line] and when it is not [eqs (17), (19) and (20)–represented by the dotted line]. (The solid line in figure 9 is the same with that in figure 7). When $\theta$ is relatively low (i.e., $\theta \leq \frac{t(2\sqrt{20} - 3)}{24}$) consumer surplus when bundling is allowed is (weakly) higher than the consumer surplus when bundling is not allowed. For high values of $\theta$ (i.e., $\theta \geq \frac{t(2\sqrt{20} - 3)}{24}$)
the above ranking of consumer surplus gets reversed. In that case an a la carte regulation will benefit consumers.

Hence, our model predicts that in the television industry, which is mainly characterized by pure bundling (PB), a regulation that would force downstream firms to unbundle is pro-competitive. The intuition for our main result is provided in the introduction. As we discuss below, we believe that this result is quite robust.

Finally, the profits of the upstream and the downstream firms are always (i.e., for any $\theta$) higher when bundling is allowed than when it is not (details are omitted). Therefore, bundling in our model does not lead to a prisoners’ dilemma. This may explain why the cable industry is opposed to an a la carte regulation. In the next section, we explore which modeling assumptions are most likely to be responsible for this result.

6 Discussion about the Robustness of our Results

6.1 The Effects of Non-Localized Competition

The most restrictive implication of our model is that competition is localized. We feel that this is the price we have to pay in order to obtain clean results. In this section, we conjecture whether our main predictions will hold and how they might be affected in a more general model of non-localized competition.

Pure bundling. First of all, when the preference for content variety parameter $\theta$ is high enough the equilibrium should be characterized by pure bundling even in a more general model. When $\theta$ is high, demand for both contents is strong and firms have little incentive to offer the contents separately. (This would require downstream firms offering the bundle at a discount, a strategy that is unprofitable when demand for both contents is sufficiently strong). As we mention in the introduction, all three effects point in the same direction, in the pure bundling case. The effect that is most likely to be affected by non-localized competition is the “change in the intensity of competition” effect. When each product competes with all other rival products (as it would be the case in a more general model) competition is more intense after an a la carte regulation is passed, than in a model of localized interactions. This is because each firm offers more products after an a la carte regulation and not just the bundle. But, clearly, this generalization will only strengthen our result.
Mixed bundling. The assumption of localized competition is less innocuous when the unregulated equilibrium is characterized by mixed bundling. In this case the “change in the intensity of competition” effect points in the opposite direction than the other two effects. When the “change in the intensity of competition” effect becomes stronger (in a more general model), we should expect the threshold of $\theta$, above which an a la carte regulation increases consumer surplus (figure 9 depicts that threshold), to increase, implying that such a regulation is less likely to have a positive effect on consumer welfare. Nevertheless, we believe that our predictions are not likely to change qualitatively. The reason is this. The “upstream” effect becomes stronger as the share of the bundle increases, relative to the share of the single purchases. This is due (as we explained in the introduction) to the fact that the upstream firms’ willingness to lower prices diminishes as fewer consumers buy their products outside the bundle. As $\theta$ increases the share of the bundle should be increasing. This suggests that the “upstream” effect may become so dominant so that an a la carte regulation will prove beneficial for the consumers even when the unregulated equilibrium exhibits mixed bundling (provided that the share of the bundle is large enough relative to the shares of the single products). Of course, this is not definite as it will depend on the nature of competition and on the strength of the “change in the intensity of competition” effect.

Prisoners’ dilemma. A number of papers in the literature [e.g. Anderson and Leruth (1993), Economides (1993) and Matutes and Regibeau (1992)] show that the possibility of bundling lowers firm profits. This is not the case in our framework. As we have indicated both upstream and downstream firms are better off when bundling is allowed than when it is not. This result can be attributed to two factors. First, the presence of upstream firms imposes some “pricing discipline” in the market. Upstream firms, when bundling is allowed, charge relatively high prices (“upstream” effect), which lowers the downstream firms’ incentives to undercut each other. This effect makes the prisoners’ dilemma less likely and it is absent from other papers in the literature. Second, competition in our model is localized and bundles do not compete head-on with each other (in the mixed bundling case). Once again, the prisoners’ dilemma is less likely to emerge. We conjecture that in a more general model of non-localized competition, firms will be likely to avoid the prisoners’ dilemma when the preference for content variety is strong enough and the share of the bundle is large relative to the shares of the single purchases. In this case the “upstream” effect would be

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15 The assumption of localized competition with regards to the likelihood of the game becoming a prisoners’ dilemma applies only to the mixed bundling case. In the pure bundling case, bundles do compete head-on with each other.
6.2 More than Two Contents

Another restriction of our model is that there are only two upstream products and two downstream firms. However, we feel that the effects that we have identified are general enough and can offer some guidance in more general environments, where for instance each upstream firm has more than one product. For example, we believe that the “upstream” effect will continue to hold as long as the bundle contains products from more than one upstream firm. Upstream firms will be less willing to lower their prices (than in a market with no bundles) because their ability to influence downstream firms’ marginal costs is diminished. The key insight from our model is that an equilibrium where the share of the bundle is large enough (or 100%) should exist when consumers value content variety a lot. In this case the “upstream” effect is strong and it is very likely to dominate the “change in the intensity of competition” effect, implying that an a la carte regulation will benefit consumers.

6.3 Advertising Revenue

Finally, our model ignores the possibility that upstream firms may also earn revenue from advertising. In reality this is a significant source of revenue. We believe that our qualitative results are not sensitive to this. The upstream firms’ advertising revenue is a function of the number of viewers. Our belief, then, is that upstream competition would intensify under an a la carte regulation, because upstream firms would have more incentives to undercut each other in order to increase (or maintain) the number of viewers. The logic behind this is similar to the intuition for the “upstream” effect. This suggests that the “upstream” effect will be even stronger when we consider revenue from advertising, making an a la carte regulation more likely to be welfare enhancing.

7 Conclusion

We develop a parsimonious model of upstream downstream competition to investigate downstream firms’ incentives to bundle. The upstream firms are content providers and the downstream firms are network operators. We use this model to shed light on the question about whether cable and satellite television companies should be forced to offer stations a la carte (i.e., to unbundle). The nature of the equilibrium depends on the strength of consumer preference for content variety. When
such a preference is: i) low, the firms do not bundle (NB), ii) medium, each firm offers the bundle and the two products separately (mixed bundling–MB) and iii) high, each firm offers only the bundle (pure bundling–PB). A regulation that forces downstream firms to unbundle leads to higher consumer surplus if and only if the share of the bundled purchases for any downstream firm is large enough (relative to the shares of the single purchases). One of our model’s prediction is that when downstream firms offer only the bundle (which is arguably the case in the television industry), then an a la carte regulation will increase consumer surplus (see figure 9).

It has also become clear from our analysis that a regulation which forces downstream firms to offer the two contents separately without prohibiting them from bundling the two contents will be ineffective, without any further regulation on prices. First, when the unregulated equilibrium is characterized by no or mixed bundling this kind of regulation obviously has no bite. Second, under the pure bundling unregulated equilibrium, downstream firms can always choose high prices for the individual contents to neutralize the effect of the regulation. In such a case all consumers, in equilibrium, will choose to buy a bundle, despite the fact that firms also offer the contents separately.

In this paper we have assumed that both contents are of equal quality. A concern is whether offering a la carte programming would hinder the ability of lesser-watched channels (i.e., channels of lower “quality”) to exist when not bundled with popular channels. This will have an adverse effect on program variety. The present model can be modified to address this issue, by introducing a quality differential between the two contents.

A Appendix I (Locations of Marginal Consumers)

Consumer $x_1$ is located at,

$$V - t \left( \frac{1}{4} - x_1 \right) - p_{1b} = V + \theta - \frac{t}{4} - p_{1B} \Rightarrow x_1 = \frac{\theta + p_{1b} - p_{1B}}{t}. \quad (21)$$

Consumer $x_2$ (assuming that $x_2$ is to the left of 1a) is located at,

$$V - tx_2 - p_{1a} = V + \theta - \frac{t}{4} - p_{1B} \Rightarrow x_2 = \frac{-4\theta + 4p_{1B} - 4p_{1a} + t}{4t}. \quad (22)$$

If $x_2$ is located between 1a and 2a, then the equation for the marginal consumer is $-x_2$. 

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Consumer \( x_3 \) is located at,
\[
V - t \left( x_3 - \frac{1}{4} \right) - p_{1b} = V - t \left( \frac{1}{2} - x_3 \right) - p_{2b} \Rightarrow x_3 = \frac{4p_{2b} - 4p_{1b} + 3t}{8t}. \quad (23)
\]

Consumer \( x_4 \) is located at,
\[
V - tx_4 - p_{1a} = V - t \left( \frac{1}{4} - x_4 \right) - p_{2a} \Rightarrow x_4 = \frac{4p_{2a} - 4p_{1a} + t}{8t}. \quad (24)
\]

Consumer \( x_5 \) is located at,
\[
V - t \left( x_5 - \frac{1}{4} \right) - p_{2a} = V + \theta - \frac{t}{4} - p_{2B} \Rightarrow x_5 = \frac{-2\theta + 2p_{2B} - 2p_{2a} + t}{2t}. \quad (25)
\]

Finally, consumer \( x_6 \) (assuming that \( x_6 \) is to the right of \( 2b \)) is located at,
\[
V - t \left( \frac{1}{2} - x_6 \right) - p_{2b} = V + \theta - \frac{t}{4} - p_{2B} \Rightarrow x_6 = \frac{4\theta - 4p_{2B} + 4p_{2b} + t}{4t}. \quad (26)
\]

If \( x_6 \) is located between \( 1b \) and \( 2b \), then the equation for the marginal consumer becomes
\[
1 - x_6 = \frac{-4\theta + 4p_{2b} - 4p_{2b} + 3t}{4t}.
\]

The consumers who are located in the interval \([x_2, x_1]\) prefer, by construction, the bundle \( 1B \) to either \( 1a \) or \( 1b \). Similarly, the consumers who are located in the interval \([x_5, x_6]\) prefer, by construction, the bundle \( 2B \) to either \( 2a \) or \( 2b \). In addition, \( x_3 \) and \( x_4 \) must be located, by construction, between \( 1b \) and \( 2b \) and between \( 1a \) and \( 2a \) respectively.\(^{16}\)

\[\text{B Appendix II (Proof of Proposition 1)}\]

\[\text{B.1 Proof - Mixed Bundling Equilibrium (MB)}\]

We first search for an equilibrium where firms offer the bundle and the individual goods separately. We solve the game backwards.

**Second stage (downstream).** The demand functions are given by (1-6). The downstream firms’ profit functions are given by,
\[
\begin{align*}
\pi_{D_1} &= (p_{1a} - r_a) d_{1a} + (p_{1b} - r_b) d_{1b} + (p_{1B} - r_a - r_b) d_{1B}, \\
\pi_{D_2} &= (p_{2a} - r_a) d_{2a} + (p_{2b} - r_b) d_{2b} + (p_{2B} - r_a - r_b) d_{2B}.
\end{align*}
\]

\(^{16}\)Furthermore, \( x_1 \) must be between \( 1a \) and \( 2b \) (on the left half of the circle); \( x_2 \) must be between \( 2a \) and \( 1b \) (on the upper half of the circle); \( x_3 \) must be between \( 1a \) and \( 2b \) (on the right half of the circle) and \( x_5 \) must be between \( 2a \) and \( 1b \) (on the bottom half of the circle). To see this, suppose, by way of contradiction, that \( x_1 \) is located between \( 1a \) and \( 2b \), on the right half of the circle. First note that, by construction, \( x_2 \) must be located to the right of \( x_1 \). Second, again by construction, the consumer at \( x_1 \) is indifferent between \( 1b \) and \( 1B \). These imply that a consumer who is located in the open interval \((x_1, x_2)\) must strictly prefer \( 1b \) to \( 1B \), since the transportation cost of \( 1B \) increases–starting from \( x_1 \)–faster than that of \( 1b \). This is a contradiction to the assumption that the consumers located in the interval with two boundaries \( x_1 \) and \( x_2 \) prefer the bundle. The same holds for the other critical points (consumers).
Each firm’s profit function is strictly concave with respect to the firm’s strategic variables.\(^{17}\) Firm \(D_1\)’s first order conditions are given by,

\[
\begin{align*}
\frac{\partial \pi_{D_1}}{\partial p_{1a}} &= -8\theta - 24p_{1a} + 4p_{2a} + 16p_{1B} + 3t + 4r_a - 8r_b, \\
\frac{\partial \pi_{D_1}}{\partial p_{1b}} &= -8\theta - 24p_{1b} + 4p_{2b} + 16p_{1B} + 3t + 4r_b - 8r_a, \\
\frac{\partial \pi_{D_1}}{\partial p_{1B}} &= 16\theta + 16p_{1a} + 16p_{2b} - 32p_{1B} - 2t + 8r_a + 8r_b.
\end{align*}
\]

Firm \(D_2\)’s first order conditions can be derived similarly. The unique and symmetric solution to the system of first order conditions is given by,

\[
p_{1a} = p_{2a} = \frac{4r_a}{5} + \frac{r_b}{5} + \frac{t}{2}, \quad p_{1b} = p_{2b} = \frac{4r_b}{5} + \frac{r_a}{5} + \frac{t}{2} \quad \text{and} \quad p_{iB} = \frac{\theta}{2} + \frac{3r_a}{4} + \frac{3r_b}{4} + \frac{7t}{16}. \quad (29)
\]

Note that the implicit requirement \(p_{ia} + p_{ib} \geq p_{iB}\) (i.e., the price of the bundle is lower than the sum of the individual prices of the goods in the bundle) holds if and only if, \(2\theta \leq r_a + r_b + \frac{9t}{4}\). If we substitute (29) into (1 & 4), (2 & 5) and (3 & 6) we obtain the stage 2-equilibrium demand functions,

\[
\begin{align*}
d_{1a} &= d_{2a} = \frac{-40\theta - 4r_a + 44r_b + 25t}{80t}, \quad d_{1b} = d_{2b} = \frac{-40\theta - 4r_b + 44r_a + 25t}{80t} \quad \text{and} \quad (30) \\
d_{1B} &= d_{2B} = \frac{8\theta - 4r_a - 4r_b - t}{8t}.
\end{align*}
\]

By substituting (29) into (21, 22, 23) we obtain the stage 2-equilibrium locations of the marginal consumers,

\[
\begin{align*}
x_1 &= \frac{40\theta - 44r_a + 4r_b + 5t}{80t}, \quad x_2 = \frac{-40\theta + 44r_b - 4r_a + 15t}{80t}, \quad x_3 = \frac{3}{8}, \\
x_6 &= \frac{40\theta - 44r_a + 4r_b + 25t}{80t}, \quad x_5 = \frac{-40\theta + 44r_b - 4r_a + 35t}{80t}, \quad x_4 = \frac{1}{8}. \quad (31)
\end{align*}
\]

For \(d_{iB} \geq 0\) we must have \(\theta \geq \frac{r_a}{2} + \frac{r_b}{2} + \frac{t}{8}\). Therefore, for bundling to arise in equilibrium, \(\theta\) must satisfy the following conditions,

\[
\frac{r_a}{2} + \frac{r_b}{2} + \frac{t}{8} \leq \theta \leq \frac{r_a}{2} + \frac{r_b}{2} + \frac{9t}{8}. \quad (32)
\]

By substituting (29) into (27) we derive the equilibrium profit functions of both downstream firms,

\[
\Pi_{D_i} = \frac{2000 tr_a - 1600\theta (r_a + r_b) + 200tr_b + 825t^2 + 32r_ar_b + 784 (r_a^2 + r_b^2) + 1600\theta^2 - 4000t^2}{3200t}. \quad (33)
\]

\(^{17}\)The \(3 \times 3\) Hessian matrix is negative definite. Therefore, the first order conditions are sufficient for a maximum.
First stage (upstream). The profit functions of the upstream firms are given by (we assume a zero marginal cost),

\[ \pi_{U_a} = (d_{1a} + d_{2a} + d_{1B} + d_{2B}) r_a \] and \[ \pi_{U_b} = (d_{1b} + d_{2b} + d_{1B} + d_{2B}) r_b. \] (34)

The solution to the first order conditions is,\(^{18}\)

\[ r_a = r_b = \frac{10\theta}{21} + \frac{5t}{28}. \] (35)

By substituting (35) into the constraints, given by (32), we derive the region of \( \theta \) so that our assumptions hold,

\[ \frac{51t}{88} \leq \theta \leq \frac{219t}{88} \text{ or approximately } 0.58t \leq \theta \leq 2.49t. \] (36)

If \( \theta < \frac{51t}{88} \), then \( d_{iB} = 0 \) and if \( \theta > \frac{219t}{88} \), then the price of the bundle is greater than the sum of the prices of the individual goods. By substituting (35) into (33) we derive the profits of the upstream firms,

\[ \Pi_{U_a} = \Pi_{U_b} = \frac{55(3t + 8\theta)^2}{14112t}. \]

The stage 2 (downstream) equilibrium prices can be obtained by substituting (35) into (29) and are given by,

\[ p_{ia} = p_{ib} = \frac{10\theta}{21} + \frac{19t}{28} \text{ and } p_{iB} = \frac{17\theta}{14} + \frac{79t}{112}. \] (37)

The equilibrium bundle demand is,

\[ d_{1B} = d_{2B} = \frac{88\theta - 51t}{168t}. \]

The bundle demand is less than or equal to \( \frac{1}{2} \) provided that \( \theta \leq \frac{135t}{88} \approx 1.534t \). Otherwise, we move to the pure bundling case where all consumers purchase the bundle (this case will be analyzed later).

The downstream equilibrium profits are given by,

\[ \Pi_{D_1} = \Pi_{D_2} = \frac{7744\theta^2 + 16713t^2 - 8976\theta t}{56448t}. \]

Next we to examine a “global” deviation on part of the upstream firms.

\(^{18}\)The second order conditions are satisfied with strict inequality. Therefore, the first order conditions are sufficient for a “local” maximum. Still, an upstream firm can deviate globally, by increasing its price drastically, so that downstream firms do not find it profitable to offer the bundle. We examine this kind of deviation next.
Checking for Profitable Deviations from the Proposed Equilibrium

Deviation by firm $U_a$. Due to symmetry, we consider only one upstream firm’s deviation. Firm $U_a$ can deviate by increasing its price $r_a$ beyond the point where $d_iB = 0$. We solve the stage 2 game assuming that no consumer purchases the bundle. It is important here to emphasize that we allow consumers to create their own bundle by purchasing both products even when a firm does not offer the bundle. Nevertheless, in this deviation, this option will not be exercised. This is because, in what follows, the bundle demand is zero when the bundle price is less than the sum of the prices of the individual goods. Therefore, it must be the case that no consumer would want to create her own bundle by paying $p_{ia} + p_{ib}$.

The stage 2 equilibrium prices are,

$$p_{ia} = p_{ib} = \frac{4r_b}{5} + \frac{r_{dev}^a}{5} + \frac{t}{2},$$

where $r_b$ is given by (35). For $d_iB$ to be zero it must be that [from (32)],

$$\frac{r_{dev}^a}{2} + \frac{r_b}{2} + \frac{t}{8} > \theta \Rightarrow r_{dev}^a > \frac{32\theta}{21} - \frac{12t}{28}.$$ 

The optimal deviation price is, $r_{dev}^a = \frac{5\theta}{21} + \frac{85t}{168} > \frac{32\theta}{21} - \frac{12t}{28}$, provided that $\theta < \frac{3297t}{4536} \approx 0.723t$. The maximum profits after deviation are given by,

$$\Pi_{U_a}^{dev} = \frac{5(17t + 8\theta)^2}{9408t}.$$

It turns out that this type of deviation is unprofitable (i.e., $\Pi_{U_a} \geq \Pi_{U_a}^{dev}$), if and only if $\theta \geq \frac{1}{8} \left(\frac{-15 + 14\sqrt{66}}{19}\right) t \approx 0.65t$.

Therefore, if

$$\theta \geq \frac{1}{8} \left(\frac{-15 + 14\sqrt{66}}{19}\right) t \approx 0.65t$$

then neither upstream firm has an incentive to deviate. Note that $\frac{1}{8} \left(\frac{-15 + 14\sqrt{66}}{19}\right) t \approx 0.65t > \frac{51t}{88} \approx 0.58t$, which has to hold in equilibrium for the bundle demand to be positive.

Therefore, if

$$\frac{1}{8} \left(\frac{-15 + 14\sqrt{66}}{19}\right) t \approx 0.65t \leq \theta \leq \frac{135t}{88} \approx 1.534t$$

then the SPNE is described by (35) and (37).

Downstream firms in this case have no incentive to deviate from (37) as long as (36) is satisfied.
B.2 Proof - Pure Bundling Equilibrium (PB)

If $\theta > \frac{135t}{88} \approx 1.534t$, then all consumers buy the bundle, half from one firm and half from the other. To find the equilibrium prices for this case set $d_{1B} = d_{2B} = \frac{1}{2}$ [from (30)] and solve for $r_a$ and $r_b$. This yields,

$$r_a = r_b = \theta - \frac{5t}{8}. \quad (38)$$

The downstream prices for the bundle are obtained by substituting (38) into (29) and are given by,

$$p_{1B} = p_{2B} = 2\theta - \frac{t}{2}. \quad (39)$$

The profits of the upstream firms are,

$$\Pi_{U_a} = \Pi_{U_b} = \theta - \frac{5t}{8} \quad (40)$$

and those of the downstream firms are,

$$\Pi_{D_1} = \Pi_{D_2} = \frac{\theta}{2} + \frac{t}{16}. \quad (41)$$

This case is actually a corner solution of the mixed bundling problem (MB) we analyzed in the previous section and there is no need to check for any deviations.

B.3 Proof - No Bundling Equilibrium (NB)

Now, we search for an equilibrium where firms do not offer the bundle. The analysis remains the same, except that $x_1 = x_2$ [see (21) and (22)] and $x_5 = x_6$ [see (25) and (26)]. As we alluded to above, consumers can create their own bundle by simply purchasing both products at the price of $p_{ia} + p_{ib}$. It will turn out, in the present case, that no consumer would find this profitable.

Second stage (downstream). The solution to the system of first order conditions is given by,

$$p_{1a} = p_{2a} = \frac{4r_a}{5} + \frac{r_b}{5} + \frac{t}{2} \quad \text{and} \quad p_{1b} = p_{2b} = \frac{4r_b}{5} + \frac{r_a}{5} + \frac{t}{2}. \quad (42)$$

The profit function of downstream firm $i$ is strictly concave with respect to $p_{ia}$ and $p_{ib}$. The first order conditions are sufficient for a “local” maximum. Nevertheless, a downstream firm can globally deviate by making the bundle available. We examine this kind of deviation later.

First stage (upstream). The solution to the system of first order conditions is given by,

$$r_a = r_b = \frac{5t}{6}. \quad (29)$$
The downstream equilibrium prices are,

\[ p_{ia} = p_{ib} = \frac{4t}{3}. \]

The equilibrium demands are given by,

\[ d_{1a} = d_{2a} = d_{1b} = d_{2b} = \frac{1}{4}. \]

The profits of the upstream firms are given by,

\[ \Pi_{Ua} = \Pi_{Ub} = \frac{5t}{12}. \]

The profits of the downstream firms are given by,

\[ \Pi_{D1} = \Pi_{D2} = \frac{t}{4}. \]

Observe that at the equilibrium downstream prices \( p_{ia} = p_{ib} = \frac{4t}{3} \), no consumer would want to create her own bundle. Even if the transportation cost is zero, the cost of a second product \( \frac{4t}{3} \) dominates the incremental benefit \( \theta \), which, in this case, is at most \( \left( \frac{-11+4\sqrt{66}}{24} \right) t \approx 0.896t \).

B.3.1 Checking for Profitable Deviations from the Proposed Equilibrium

Deviation by firm \( D_1 \). Fix the upstream prices at \( r_a = r_b = \frac{5t}{6} \) and the prices of \( D_2 \) at \( p_{2a} = p_{2b} = \frac{4t}{3} \). Firm \( D_1 \) can deviate by making the bundle available. The optimal deviation prices are given by,

\[ \begin{align*}
p_{1a}^{dev} = p_{1b}^{dev} &= \frac{4t}{3} \quad \text{and} \quad p_{1B}^{dev} = \frac{\theta + 27t}{16}.\end{align*} \]

First note that the prices of the individual goods are the same before and after the deviation. It turns out that such a deviation is always profitable, provided that the demand for the bundle, which is \( d_{1B}^{dev} = \frac{24\theta - 23t}{24t} \), is positive. Therefore, such a deviation is not profitable if and only if \( d_{1B}^{dev} = \frac{24\theta - 23t}{24t} \leq 0 \Rightarrow \theta \leq \frac{23t}{24} \approx 0.96t \). It will turn out that this constraint is not binding.

Deviation by firm \( U_a \). Fix \( r_b = \frac{5t}{6} \). Firm \( U_a \) deviates, by lowering its price, so that downstream firms find it optimal to offer the bundle. For \( d_{iB} \) to be greater than zero, it must be that \( [\text{from (32)}] \),

\[ \frac{r_a^{dev}}{2} + \frac{r_b}{2} + \frac{t}{8} < \theta \Rightarrow r_a^{dev} < 2\theta - \frac{r_b}{4} = 2\theta - \frac{13t}{12}. \]

The optimal price and profits for the deviating firm are,

\[ r_a^{dev} = \frac{5\theta}{11} + \frac{5t}{24} \quad \text{and} \quad \Pi_{Ua}^{dev} = \frac{5(11t + 24\theta)^2}{12672t}. \]
The optimal deviation price $r_{a}^{\text{dev}} = \frac{5\theta}{11} + \frac{5t}{24}$ is less than $2\theta - \frac{13t}{12}$ provided that $\theta \geq \frac{341t}{408} \approx .836t$.

It turns out that this type of deviation, without considering the price constraint, is unprofitable (i.e., $\Pi_{Ua} \geq \Pi_{Ua}^{\text{dev}}$), if and only if $\theta \leq \left(\frac{-11 + 4\sqrt{66}}{24}\right)t \approx .896t$. Hence, the price constraint is not binding, which implies that this deviation is unprofitable if and only if,

$$\theta \leq \left(\frac{-11 + 4\sqrt{66}}{24}\right)t \approx .896t.$$

Based on the analysis so far we can conclude that there are two equilibria when,

$$\frac{1}{8}\left(-15 + 14\sqrt{66}\right) t \approx .65t \leq \theta \leq \left(\frac{-11 + 4\sqrt{66}}{24}\right)t \approx .896t,$$

one with mixed bundling (MB) and the other with no bundling (NB). Moreover, the first one is the Pareto optimal equilibrium (from the perspective of the two upstream firms) provided that $\theta \leq \frac{1}{8}\left(-3 + \frac{14\sqrt{66}}{11}\right)t \approx .917t$ and consequently it will be the one preferred by these firms. This is a property of supermodular games, when the payoff to a player is increasing in the strategy of the other player, as it is the case at the upstream stage in our model [see Vives, Remark 14, p.34].

We assume that upstream firms are able to coordinate on the better equilibrium. Therefore, the no bundling equilibrium (NB) emerges if and only if $\theta \leq \left(\frac{-11 + 4\sqrt{66}}{24}\right)t \approx .896t$.

References


