MERGERS, DIVERSIFICATION AND FINANCIAL INTERMEDIATION

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Abstract. This work presents an equilibrium model of diversification through merger formation. Due to moral hazard problems, poorly capitalized firms are credit rationed and may seek to alleviate the incentive problem (and thereby raise external funds) by either merging, employing a monitor or a combination of the two. Within this setting, the effects on merger activity of different kinds of capital tightening are studied. In particular, credit crunches, collateral squeezes and savings squeezes are analyzed. The main results are that diversifying merger activity increases during times of economic expansion and is positively related to aggregate economic activity, business incorporations and easing of access to credit (both interest and non-interest terms of credit). Furthermore, the model offers a rationale for diversification that is immune to the diversification neutrality result and furthermore, explains why diversified companies trade at a discount relative to their non-diversified counterparts.

Keywords: Mergers, merger waves, diversification, diversification discount, financial intermediation, capital tightening.

JEL Classification: L16, G34.

1. Introduction

Corporate diversification through merger has a long history, both as a fact of life and as a subject of analysis by financial economists. Received wisdom is less than favorable in judging the benefits of diversification, for broadly two reasons. The first may be termed the diversification neutrality result and poses that any diversification possibilities that corporations might have, will, in a perfect capital market, already have been exhausted through shareholders’ individual portfolio choices. The second is the so-called diversification discount, i.e. the finding that diversified corporations have, during some periods, traded at a discount relative to their non-diversified counterparts. Together, these two findings have prompted scholars to conclude that corporate diversification is at best a neutral, but more likely, a value destroying strategy.1

Despite these concerns, corporate diversification continues to be strong, a finding that should prompt further study into the causes and consequences of diversification. The current paper seeks to fill this gap in the theory by showing how diversification may be useful despite the diversification neutrality result and that corporate diversification may be optimal and

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1A leading corporate finance textbook even goes as far as characterizing diversification as one of three main “Dubious Reasons for Mergers” (see Brealey and Myers, 2003, p. 934).
yet consistent with diversified firms trading at a discount relative to non-diversified firms. Furthermore, by embedding the analysis of diversification in an equilibrium framework, it is also possible to address two further issues. First, it is possible to characterize firms that choose to diversify. Second, it is possible to derive the extent of diversification in the economy. As a consequence of this last finding, it can be determined under which conditions conglomerate merger activity should be prevalent and thereby address the timing of mergers and merger waves.

The current work thus contributes to two hitherto separate literatures, namely that on corporate diversification and the diversification discount and to that on merger waves. Accordingly, the following two subsections will briefly outline existing work on those two literatures and place the current work within them.

1.1. The Diversification Debate. The central question of the diversification debate is why corporations are active in seemingly disparate lines of business. At face value, diversification can be explained by the fact that when pooling income streams that are less than perfectly positively correlated, the resulting income stream is less volatile than were the constituent income streams. Such a reduction of riskiness could potentially be beneficial to shareholders. This view of corporate diversification is challenged by numerous scholars, such as Levy and Sarnat (1970) and others. Their basic insight is that any reduction in risk that a firm may achieve through diversification can be replicated by the individual shareholders through an appropriately chosen portfolio. Shareholders may even achieve such diversification at a lower cost than the firm can. Loosely speaking, what is being criticized by this literature is diversification as an activity that is pursued because it directly benefits shareholders by reducing their exposure to risk, or in other words, a pure risk-reduction motive for corporate diversification.

Beginning with Lewellen (1970), numerous authors consider another rationale for diversification. These include Higgins (1970), Lintner (1971), Rubinstein (1973), Melnik and Pollatschek (1973), Higgins and Schall (1975) and Lee (1977). According to this strand of literature, the view of diversification as a means to decrease shareholders’ exposure to risk is unnecessarily narrow in that corporate diversification and the resulting decrease in riskiness can increase the combined entity’s debt capacity. I will term this the debt capacity motive for diversification. With an increase in debt capacity, the shareholders may benefit indirectly. The main problem with the debt capacity motive is that it suffers from some of the same drawbacks as that of the pure risk-reduction motive. Specifically, this strand of literature has ignored the fact that in perfectly functioning capital markets, firms’ access to credit (and hence their debt capacity) should be constrained only by the value of their projects. In other words, under the same conditions where the diversification neutrality result has bite, increasing debt capacity through diversification should not be a concern in the first place.

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2 A conglomerate merger is best viewed as a combination of two separate legal entities into one single entity, further characterized by the fact that the two firms are not active in the same industry, broadly defined, or vertically related by being active in the same supply chain. It should be noted that there are immense difficulties in empirically distinguishing pure conglomerate (or diversifying) mergers from vertical mergers (see e.g. Goudie and Meeks, 1982, who provide a description of firms engaged in diversifying mergers). Discrimination between the two kinds of mergers must be based on a case by case review which is prohibitive for large data sets. For a review of descriptive work on diversifying firms, see also Montgomery (1994) and references therein.

3 Actually, Levy and Sarnat (1970) themselves concede that a debt capacity motive may exist.

Another apparent problem with diversification is the finding that diversified firms trade at a discount relative to their non-diversified counterparts. A number of authors describe the existence of such a diversification discount, e.g. Lang and Stulz (1994), Berger and Ofek (1995, 1996), Servaes (1996), Lins and Servaes (1999) and Laeven and Levine (2004). One problem with interpreting the existence of a diversification discount is one of causality. If indeed diversified firms trade at a discount, is that prima facie evidence that diversification per se destroys value? In fact, the causality could be reversed if one finds that what makes firms trade at a discount is also what makes them diversify. In a nutshell, what must be recognized is that firms choose to diversify, i.e. the diversification decision itself is endogenous.

A number of recent contributions make exactly this point. Lamont and Polk (2001), Graham et al. (2002), Maksimovic and Phillips (2002), Campa and Kedia (2002), Kruse et al. (2002), Villalonga (2004a, 2004b), Yang (2005), Akbulut and Matsusaka (2005) and Pal and Bohl (2005) all revisit the value of diversified firms while explicitly taking into account the endogeneity of the diversification decision. Their results suggest that when properly acknowledging this endogeneity, the diversification discount may disappear altogether or even turn into a premium. Similarly, their work also suggests that there is a strong element of self-selection to the group of diversified firms. In particular, such firms tend to do poorly prior to diversification, are more heavily leveraged and are generally more liquidity constrained. To the best of my knowledge, Cabral (2003) is the only theoretical contribution that explicitly considers the endogeneity of the diversification decision. He does so in a model of reputation.

1.2. Aggregate Merger Activity. One of the most salient features of M&A activity is that it displays a marked wave pattern which roughly follows the business cycle. Specifically, merger activity has been found to be positively related to industrial output (Nelson, 1959, Gort, 1969, Mitchell and Mulherin, 1996), business incorporations (Nelson, 1959) and to decreases in interest rates (Melicher et al. 1983, Golbe and White, 1988, Weston et al. 1990). Weston et al. (1990) and Harford (2004) show that merger activity is positively related to easing of financing constraints (i.e. easing of non-interest rate terms of credit), mirroring results of Kashyap et al. (1994) and Lown et al. (2000) in their studies of aggregate activity and credit constraints. Last, Shleifer and Vishny (1992) argue that increases in collateral values may prompt increases in merger activity.5

The literature has, until recently, been characterized by a lack of theoretical explanations for these phenomena.6 Financial theory has mostly taken a single firm as the object of analysis without embedding the merger formation decision in an equilibrium environment. The industrial organization literature has in turn focused almost exclusively on merger formation within explicit oligopoly models, which have turned out to be somewhat unsuitable for the analysis of merger waves and their possible dependence on financial variables.

In view of this lack of theory, recent developments have been encouraging. A number of theories have been proposed, each focusing on different aspects of M&A activity. Jovanovic and Rousseau (2004) focus on mergers as responses to reorganization possibilities brought about by changes in technology. Rhodes-Kropf and Viswanathan (2004) show how over

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5 For a review of the early empirical literature, see Weston et al. (1990).
6 Brealy and Myers (2003) consider merger waves to be one of the major unresolved puzzles of finance theory.
valuation may lead to increases in merger activity. These theories are non-strategic in the sense that no merger is, per se, prompted by other mergers in the industry or the economy. Toxvaerd (2004) considers a model in the alternative category, focusing on strategic aspects of the merger decision and on how such strategic considerations can prompt firms to decide to merge at the same time.

The current paper adds to the first (non-strategic) strand of literature by identifying another channel through which aggregate variables can prompt increases in M&A activity. In the economy under consideration, poorly capitalized firms rely on financial intermediaries in order to raise funds for projects. By merging with unrelated firms, a firm may also boost its debt capacity. In turn, this means that the equilibrium extent of merger activity is a function of the amount of funds available from financial intermediaries. In equilibrium, when intermediary capital is plentiful, more firms will become active in the economy, many of them becoming so by forming conglomerates.

1.3. Overview and Results. The present model builds on work by Holmström and Tirole (1997). They consider the effects of capital tightening on an economy in which cash-poor firms rely on financial intermediation to finance their projects. I enrich their model by allowing firms either to seek the assistance of a financial intermediary, to engage in a diversifying merger, or (if they so desire) any combination of the two. Assuming that projects of a merged firm are independent, I draw on a diversification result found in Diamond (1984) and show that by diversifying, a firm may relax incentive constraints and thus facilitate funding.

In a nutshell, the aim of the analysis is to consider how shifts in aggregate demand and supply of capital affect funding possibilities for cash-poor firms and in turn how the firms’ incentives to engage in diversifying mergers are affected by such changes in funding possibilities.

Two distinct scenarios are considered. In the first, the supply of uninformed capital is infinitely elastic and the return on this capital is thus exogenously given. In this scenario, equilibrium is completely characterized by equilibrium in the market for intermediary capital. I find that an increase in the amount of capital held by the intermediary sector (i.e. the opposite of a credit crunch) decreases intermediaries’ rate of return. In turn, this increases economic activity in the real sector of the economy. This increase is brought about by an increase in the amount of diversified firms. In the second scenario, the supply of uninformed capital is modeled by means of a standard, increasing supply function and equilibrium must obtain in both markets for capital, informed and uninformed. The two kinds of capital are found to be gross substitutes which introduces the possibility of multiple equilibria. I find that in any stable equilibrium, credit crunches, savings squeezes and collateral squeezes increase both intermediaries’ returns and interest rates. This means that when credit and savings are plenty, or when collateral values increase, rates of return and interest rates decrease. In consequence, economic activity increases, as does the extent of merger activity. The comparative statics predictions of the model are therefore robust across scenarios. Furthermore, the predictions are consistent with empirical findings.

Last, because the firms that choose to diversify are those with lowest net worth, namely those firms that also rely on costly financial intermediation, the average value of diversified firms is found to be lower than that of richer non-diversified ones that access the financial markets directly.

Section 2 contains a presentation of the basic model and the derivation of a number of
intermediate results needed for the analysis in subsequent sections. Building on the results contained in Section 2, in Section 3 I discuss the parameterization of the model and derive expressions for and properties of the aggregate demand functions for informed and uninformed capital respectively. In Section 4, equilibria of the model are characterized under two alternative scenarios, that of infinitely elastic supply of uninformed capital and that of elastic supply. This section also presents comparative statics results and discusses their relation to stylized facts of merger activity. In Section 5, the equilibrium value of firms is discussed and contrasted with the literature on the diversification discount. Last, Section 6 offers a discussion and concluding remarks.

2. The Model

The economy under consideration is populated by three separate classes of risk neutral agents. There is a continuum of firms making up the real sector of the economy. These firms can each undertake a project requiring up front investment $I > 0$. Each firm is characterized by its initial holding of assets $K \geq 0$. The amount of assets $K$ can be thought of either as cash or as the monetary value of collateral. Assume that the assets are uniformly distributed along an interval $[0, L]$ of firms, where $L < I$. If a project is undertaken, the outcome depends on the unobservable effort exerted by the entrepreneur managing it. Specifically, the entrepreneur can choose high, intermediate or low effort, leading to a probability of success $p_H$ if effort is high and $p_L$ if effort is intermediate or low, where $p_L < p_H$. Why switching between intermediate and low effort does not change the probability of success will become apparent below, where the role of intermediaries is discussed. For convenience, define $\Delta p \equiv p_H - p_L$.

In case of success, the investment returns $R > 0$, while a failed project returns zero. To introduce moral hazard into the firm’s problem, assume that the entrepreneur enjoys private benefit $S > 0$ from exerting low effort, $s < S$ with $s > 0$ from exerting intermediate effort and zero from exerting high effort. The private benefit can be thought of as an opportunity cost of working on the project. Alternatively, the choice of the agent can be thought of as the choice between three different projects, a good project $G$, an intermediate project $s$ and a bad project $S$. The table below summarizes the three versions of the project in terms of private benefit $z$ and success probability $p$:

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<tr>
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<td>$p_L$</td>
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<tr>
<td>$z$</td>
<td>0</td>
<td>$S$</td>
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It should be noted immediately that the firm will always choose an $S$ project over an $s$ project, since it yields higher private benefit without lowering the probability of success.

Since a firm’s assets are such that $K < I$, it needs to raise an amount $I - K > 0$ externally in order to realize the project. Assuming that the choice of effort is unobservable to outsiders, there is a moral hazard problem which must be taken into account when deriving the optimal contract vis-a-vis outside investors. Since moral hazard is trivial when the agent is risk neutral, a limited liability constraint is imposed such that the agent must have a non-negative return even in case of failure.

Turning to the financial sector, a continuum of uninformed investors invest in projects in return for adequate compensation. The market for uninformed capital is assumed to be competitive. Furthermore, these investors can only observe the outcome of the project, not the agent’s effort. As regards the aggregate supply of uninformed capital, two scenarios will
be considered. In the first, there is an infinite amount supplied at rate $\gamma$. In the second, uninformed capital is supplied according to an upward-sloping supply function $S(\gamma)$.

Last, there are informed investors (or monitors). The informed investors can be thought of as financial intermediaries such as venture capitalists or large banks. These investors can, at a personal and unverifiable cost, inspect a firm’s project and reduce the private benefit from shirking. Specifically, the monitor can, at cost $c > 0$, rule out the $S$ project, effectively reducing the monitored firm’s options to the $G$ and $s$ projects. Since monitoring activity is unobservable, the informed investors’ monitoring activity is subject to moral hazard, a fact which is reflected in the monitors’ incentive scheme. Only monitors have access to the monitoring technology and are assumed to operate in a competitive market. It should be noted at this stage that the the monitors may, if they so wish, invest in the market for uninformed capital on the same terms as the uninformed investors.

The market for intermediation deserves some further discussion. First note that since monitoring activity is subject to moral hazard, the problem of providing incentives to monitor is very much like that of providing the firm with incentives to behave diligently. In the same way that firms may relax financing constraints by diversifying, a financial intermediary may diversify through its choice of projects to monitor. This is exactly the case considered by Diamond (1984), where he shows that if there are no overload costs to monitoring projects, the intermediary can diversify away all risk by monitoring an unbounded number of firms. This would have the effect that the intermediary could get away with monitoring firms without having itself a financial stake in the monitored firms. Since an integral part of the current analysis is to determine the effects of aggregate capital held by the intermediary sector on aggregate merger activity, it is assumed for simplicity, like in Holmström and Tirole (1997), that projects monitored by an intermediary are perfectly correlated. This means that the amount of monitoring activity is bounded by the intermediary sector’s asset holdings.

Denote by $K_F$ and $K_M$ the aggregate amounts of firm capital and monitoring capital respectively. For later use, note that the aggregate amount of firm capital is given by

$$K_F = \int_0^L KdU(K) = L^22^{-1}$$

In the full version of the model, firms are allowed to either (i) stay independent and seek direct finance, (ii) stay independent and seek indirect finance, (iii) merge and seek direct finance, (iv) merge and seek indirect finance of both projects and (v) merge and seek indirect finance of only one of the projects.

In the following subsections, the optimal financial contracting arrangements between firms and outside investors will be characterized and the related capital requirements for each type of financing will be determined. Once the capital requirements of each firm, as a function of its net worth, has been determined, the aggregate demands for informed and uninformed capital can be derived. These will in turn be used to describe equilibrium patterns of merger activity and to derive comparative statics results.

2.1. Direct Finance. First, consider the financial contracting problem faced by a single firm seeking to raise funds directly on the market for uninformed capital. As noted above,

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7 See Diamond (1984) and Holmström and Tirole (1997) for a thorough discussion of different interpretations of intermediaries' role in monitoring firms.

8 For a more detailed exposition of the basic building blocks of the model, i.e. the results contained in sections 2.1, 2.2 and 2.3, see Tirole (2005).
the firm will never choose an s project if his project choice is unobservable. The relevant options for the firm are thus given by the following table:

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<tr>
<td>p</td>
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<td>pL</td>
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<td>z</td>
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I will start by considering the entrepreneur’s incentives when writing the optimal debt contract.9 Letting \( R_F \) and \( R_U \) denote the firm’s and the (uninformed) investor’s return in case of success, a sharing rule is given by

\[
R_F + R_U = R
\]

Assuming that it is indeed optimal to induce the agent to exert high effort (choosing probability \( p_H \)), the incentive compatibility constraint is \( p_H R_F \geq p_L R_F + S \), or

\[
R_F \geq \frac{S}{\Delta p}
\]

Define pledgeable income as the maximum amount that can be pledged to investors while maintaining the right incentives for the agent. This amount is given by

\[
p_H \left( R - \frac{S}{\Delta p} \right)
\]

Letting \( \gamma \) be the rate of return on the market for uninformed capital, it follows immediately that the firm obtains finance if and only if

\[
p_H \left( R - \frac{S}{\Delta p} \right) \geq \gamma (I - K)
\]

where \( p_H R_U = \gamma (I - K) \) is the minimum return to uninformed investors. Define the following threshold for net worth:

\[
A(\gamma) \equiv I - \frac{p_H}{\gamma} \left( R - \frac{S}{\Delta p} \right)
\]

For \( K < A(\gamma) \), the firm cannot raise external finance directly on the market for uninformed capital, while it can for \( K \geq A(\gamma) \). Last, note that \( A(\gamma) \) is an increasing function of \( \gamma \).

Summing up, the existence of a moral hazard problem effectively divides the firms into two distinct groups. Poorly capitalized firms cannot rely on direct funding from uninformed investors since the return that these must be credibly promised to be able to break even is not compatible with the firms’ incentives to exert high effort.

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9With a slight abuse of notation, \( R_F, R_M \) and \( R_U \) will denote the rewards to the firm, the monitor and the uninformed investors respectively, across scenarios. Although the magnitude of these rewards will vary across the sections of the paper, this notation is only employed in intermediate calculations and should cause no confusion in context.
F. Toxvaerd

2.2. Indirect Finance. Next, the potential scope for intermediated financing of a single firm is considered. In order to do so, I revert to the setup with three versions of the project as given in the table below:

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<th></th>
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<td>z</td>
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I assume that a monitor or financial intermediary can rule out the S project at personal (and unverifiable) cost $c > 0$. To induce the monitor to monitor, he must be provided with the right incentives. Letting $R_M$ denote the return to the monitor in case of success, the relevant incentive compatibility constraint is given by

$$R_M \geq \frac{c}{\Delta p}$$

which is akin to the incentive constraint of the agent. Given that the monitor monitors, the agent’s incentive constraint is

$$R_F \geq \frac{s}{\Delta p}$$

and the sharing rule is the following three-way split:

$$R_F + R_U + R_M = R$$

Monitoring is useful because it reduces the agent’s opportunity cost of choosing the $G$ project, not because the private benefit changes from $S$ to $s + c$.

Since

$$p_H R_M - c = \frac{p_H c}{\Delta p} - c = \frac{p_L c}{\Delta p} > 0$$

the monitor earns a strictly positive rent from monitoring. But the monitor’s incentives are provided solely by the expectation of receiving $R_M$ in case of success and consequently, the firm can regulate the investor’s return by requiring him to make some contribution $I_M(\beta)$ towards the funding of the project. Defining the return on monitoring capital by

$$\beta = \frac{p_H R_M}{I_M(\beta)}$$

it follows from the monitor’s incentive constraint that

$$I_M(\beta) \geq \frac{p_H c}{\beta \Delta p}$$

Since $\gamma$ is the rate of return on the market for uninformed capital, the minimum acceptable return on monitoring capital is implicitly given by

$$\frac{p_H c}{\Delta p} - c = \gamma I_M(\beta)$$

or, rewriting, by

$$\beta = \gamma \frac{p_H}{p_L} > \gamma$$
In turn, this implies that the monitor must contribute at least

\[ I_M(\beta) \geq \frac{p_L c}{\gamma \Delta p} \]

towards the funding of the project.\(^{10}\) Last, since monitoring capital is expensive (\(\beta > \gamma\)), it is easy to see that an entrepreneur will avoid monitoring whenever possible and in case monitoring is essential, he will use the minimum possible amount of monitoring capital.

Assuming that the monitor is induced to monitor (and thus reduces the agent’s private benefit to \(s\)) and substituting for the incentive constraints, yields pledgeable income

\[ p_H \left( R - \frac{s + c}{\Delta p} \right) \]

Since the expected return to uninformed investors must be at least \(p_H R_U = \gamma (I - K - I_M(\beta))\), their individual rationality constraint is

\[ p_H \left( R - \frac{s + c}{\Delta p} \right) \geq \gamma (I - K - I_M(\beta)) \]

Thus the cut-off level of net worth above which the firm can obtain funding with the aid of an intermediary is

\[ B(\gamma, \beta) \equiv I - I_M(\beta) - \frac{p_H}{\gamma} \left( R - \frac{s + c}{\Delta p} \right) \]

Again, for \(K \geq B(\gamma, \beta)\) the firm can obtain finance with the assistance of a monitor, while for \(K < B(\gamma, \beta)\), it cannot. Last, \(B(\gamma, \beta)\) is increasing in both arguments.

Note that as long as \(s + c \geq S\), employing a monitor actually decreases the firm’s debt capacity but that the decrease in pledgeable income is compensated by the monitor’s contribution towards the project.

Summing up, employing a monitor allows some poorly capitalized firms without access to direct finance the opportunity to fund their projects with a mixture of uninformed capital and informed (monitoring) capital. Under the parameterization outlined below, no firm would, if given the choice, opt for informed capital, as monitoring comes at an increased price vis-à-vis uninformed capital.

2.3. Diversification by Merger and Direct Finance. Next, I consider the financial contracting problem faced by a merged (diversified) firm. In order to do so, I now revert to the version of the model with only the \(G\) and the \(S\) projects available (again, the \(s\) project is a strictly dominated choice) and consider two firms with assets \(K'\) and \(K''\) respectively. For simplicity, I define \(K = (K' + K'')/2\), so the firms have an average net worth \(K\) each and consider a merger between two such firms. It now has two projects requiring a total of \(2I\) and net worth \(2K\). I assume that the two projects managed by the merged unit are independent.\(^{11}\) To ensure high effort on both projects, it must be worthwhile for the manager to give up \(2S\) (which he would enjoy by shirking on both projects). The relevant quantities are summarized in the following table, where \(p\) is now the probability of success on both projects and \(z\) is the corresponding private benefit from different project choices:

\(^{10}\)With such a contribution, a monitor’s net gain from monitoring is given by \(p_H R_M - c - \gamma I_M(\beta) = 0\).

\(^{11}\)It will not be specified how the two owners of a merged firm divide the return to their investments. It can either be assumed that they divide it according to a sharing rule determined by their relative contributions, or that one of the managers buys out the other from the outset. What is important is that after the merger they act as if they were one.
In all generality, the returns to the firm from two, one and no successful projects must be specified. As will now be shown, there is no loss of generality in restricting attention to contracts which specify a positive return to the firm only in case both projects are successful. In particular, the following result holds:

**Lemma 1.** The optimal debt contract pays the entrepreneur \( R_F \) if there is success on both projects and zero otherwise.

**Proof.** The optimal contract that induces the agent to choose \((G,G)\) is sought. Three different possibilities must be considered: The agent chooses \((G,G)\), he chooses \((S,G)\) or \((G,S)\), or he chooses \((S,S)\). Let \( R_i \) denote reward if there is success on \( i = 0,1,2 \) projects.

First observe that \( R_0 = 0 \) is trivially optimal (otherwise it would be more expensive to give incentives to choose \((G,G)\) projects). This reduces the set of incentive compatibility constraints to the following inequalities:

\[
\begin{align*}
p_H^2 R_2 + 2p_H(1 - p_H)R_1 & \geq p_H p_L R_2 + (p_H + pl - 2p_H p_L)R_1 + S \\
p_H^2 R_2 + 2p_H(1 - p_H)R_1 & \geq p_H^2 R_2 + 2p_L(1 - p_L)R_1 + 2S
\end{align*}
\]

The first inequality ensures that \((G,G)\) dominates \((G,S)\) or \((S,G)\). The second, \((2)\), ensures that \((G,G)\) dominates \((S,S)\).

Next, I consider a contract \( \{R_2, R_1, 0\} \) that satisfies the two incentive constraints. It must be shown that there exists a contract \( \{R_F, 0, 0\} \) that also satisfies the incentive constraints and that

\[
p_H^2 R_F = p_H^2 R_2 + 2p_H(1 - p_H)R_1
\]

which just states that the agent is indifferent between these two contracts. It is useful to rewrite the constraints as

\[
\begin{align*}
p_H R_2 + (1 - 2p_H)R_1 & \geq \frac{S}{\Delta p} \\
p_H R_2 + (1 - p_H - p_L)R_1 & \geq \frac{2S}{\Delta p}
\end{align*}
\]

Next, I consider changes \( \Delta R_2 > 0 \) and \( \Delta R_1 < 0 \) that leave \((1)\) unaffected, i.e. such that \( p_H \Delta R_2 + (1 - 2p_H) \Delta R_1 = 0 \), or

\[
\Delta R_2 = \left(\frac{2p_H - 1}{p_H}\right) \Delta R_1
\]

Pledgeable income is given by

\[
2p_H^2 R + 2p_H(1 - p_H)R - p_H^2 R_2 - 2p_H(1 - p_H)R_1
\]

The impact on pledgeable income brought about by the changes \( \Delta R_2 \) and \( \Delta R_1 \) is thus

\[
-p_H^2 \Delta R_2 - 2p_H(1 - p_H)\Delta R_1
\]
To determine the sign of this change, I substitute for $\Delta R_2$ to get

$$-p_H^2 \left( \frac{2p_H - 1}{p_H} \right) \Delta R_1 - 2p_H(1 - p_H) \Delta R_1 = -\Delta R_1 p_H > 0$$

Thus these changes increase pledgeable income. It must be checked that such a change in $R_2$ and $R_1$ that leaves (2) unaffected also increases pledgeable income. It must be the case that $(p_H + p_L)\Delta R_2 - 2(p_H + p_L - 1)\Delta R_1 = 0$, or

$$\Delta R_2 = \left( \frac{2(p_H + p_L - 1)}{p_H + p_L} \right) \Delta R_1$$

Thus the change in pledgeable income is

$$-p_H^2 \left( \frac{2(p_H + p_L - 1)}{p_H + p_L} \right) \Delta R_1 - 2p_H(1 - p_H) \Delta R_1 = -\Delta R_1 \left( \frac{2p_H p_L}{p_H + p_L} \right) > 0$$

This completes the proof.

Thus it is not restrictive to consider contracts where the agent is paid only when there is success on both projects. The constraint ensuring that the agent prefers to work on both projects than to shirk on both projects implies the constraint that the agent prefers to work on both projects than to work on only one project. To see this, rewrite the two conditions (1)-(2) as

$$R_F \left( \frac{p_H + p_L}{2} \right) \geq \frac{S}{\Delta p}$$

$$R_F p_H \geq \frac{S}{\Delta p}$$

Since $p_H > p_L$, the second constraint is satisfied if the first one is. The relevant incentive compatibility constraint is thus

$$R_F \geq \frac{2S}{(p_H + p_L)\Delta p}$$

The expected payoff to the agent is then

$$p_H^2 R_F = \left( \frac{p_H}{p_H + p_L} \right) \frac{2p_H S}{\Delta p} = \frac{2p_H(1 - d)S}{\Delta p}$$

where

$$d \equiv \frac{p_L}{p_H + p_L} \in [0, 1/2]$$

is a measure of the effectiveness of diversification.

Since uninformed investors must earn at least $p_H R_U = \gamma (2I - 2K)$, the two projects get jointly funded only if

$$2p_H \left( R - \frac{(1 - d)S}{\Delta p} \right) \geq \gamma (2I - 2K)$$

Again, this defines a threshold of average net worth per project:

$$C(\gamma) \equiv I - \frac{p_H}{\gamma} \left( R - \frac{(1 - d)S}{\Delta p} \right)$$
For \( K < C(\gamma) \), the merged firm cannot obtain direct funding, while it can for \( K \geq C(\gamma) \). Last, \( C(\gamma) \) is an increasing function of \( \gamma \).

Note that in contrast to the case in which monitoring is employed, diversification by merger actually boosts the firms’ joint debt capacity. This is because the merger makes it cheaper to induce high effort and thereby reduces the firms’ incompressible share of the merged company. Furthermore, this expansion in debt capacity comes at no additional cost in terms of rents left to outside investors.

2.4. Diversification by Merger and Indirect Finance. Now the model is extended in a straightforward way, by allowing firms to merge and to employ monitors on either or both of a merged firm’s projects. The table below shows the probabilities of success \( p \) on both projects and private benefits \( z \) from different project combinations. For example, if the agent chooses \((G, s)\), the probability of success of both projects is \( p = p_H p_L \) and the agent earns benefit \( z = s \) from the project which he shirks on.

<table>
<thead>
<tr>
<th></th>
<th>( G, G )</th>
<th>( G, S )</th>
<th>( G, s )</th>
<th>( S, S )</th>
<th>( S, s )</th>
<th>( s, s )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( p )</td>
<td>( p_H^2 )</td>
<td>( p_H p_L )</td>
<td>( p_H p_L )</td>
<td>( p_L^2 )</td>
<td>( p_L^2 )</td>
<td>( p_L^2 )</td>
</tr>
<tr>
<td>( z )</td>
<td>( 0 )</td>
<td>( S )</td>
<td>( s )</td>
<td>( 2S )</td>
<td>( S + s )</td>
<td>( 2s )</td>
</tr>
</tbody>
</table>

As in the previous sections, conditions on net worth for the merged unit that ensure direct or indirect finance will now be derived. While there are now more combinations to consider, the analysis is only slightly complicated by this. It is useful to recall that when a merged firm approaches the market for uninformed capital, the optimal contract pays it \( R_F \) if there is success on both projects and zero otherwise. I now consider a merged firm with average per project net worth \( K \).\(^{12}\) As noted above, the monitor can (if provided the incentives to do so) identify and rule out \( S \) projects at personal cost \( c > 0 \) per project. Thus, it costs him \( c \) to rule out the \((S, S)\) project combination and \( 2c \) to rule out the \((S, s)\) or \((s, S)\) project combinations. When writing the optimal contract with monitoring, the following results are useful:

**Lemma 2.** (i) The optimal debt contract with monitoring on both projects (complete monitoring) pays the agent and the monitor \( R_F \) and \( R_M \) respectively in case of success of both projects and zero otherwise. (ii) The optimal debt contract with monitoring on only one project (partial monitoring) pays the agent \( R_F \) in case of success of both projects and zero otherwise, while the monitor earns \( R_M \) in case of success on the monitored project.

**Proof.** (i) Given that the monitor is provided incentives to monitor both projects, the agent is facing the same problem as a merged firm in the absence of monitoring possibilities, with \( s \) substituted for \( S \). Next, consider the monitor’s incentives to monitor. When writing the optimal contract for the monitor, the following three possibilities must be compared: monitor both projects, monitor just one project or monitor no projects. It is immediately clear that the problem of providing the right incentives for the monitor to monitor is essentially the same as the problem of providing incentives to the entrepreneur to choose two good projects. The result follows immediately.

(ii) In the absence of monitoring possibilities, the merged firm had to compare the following three options: \((G, G)\), \((G, S)\) or \((S, G)\) and \((S, S)\). Now consider the following setup: A

\(^{12}\) As before, \( K = (K' + K'')/2 \) where \( K' \) and \( K'' \) are the merging firms’ respective net worth.
monitor is assigned to monitor a single project, say project 1.\footnote{Note that it is indeed optimal to assign the monitor to a specific project under partial monitoring. This is because if the monitor can freely choose which project to monitor and keep his choice secret, he would be rewarded in case of success on any of the two projects. This would involve rewarding him in states of the world where he has chosen project 1, say, has shirked, but there was success on project 2. He would thus be rewarded for an outcome which he has no influence on.} When writing the contract with partial monitoring, the following four alternatives are available: the agent works on both, the agent shirks on both, the agent works on the monitored project and shirks on the non-monitored project and last, he shirks on the monitored project and works on the non-monitored project. It is immediately clear that if the agent were to shirk on a single project, he would always choose to shirk on the non-monitored project. This leaves three relevant alternatives: work on both (yielding zero private benefit), shirk on both (yielding private benefit \( S + s \)) or work on the monitored and shirk on the non-monitored project (yielding private benefit \( S \)). But note that the problem is then essentially the same as that under complete monitoring, except that the value of shirking is reduced only for one instead of two projects (namely the monitored one). The result follows immediately.

While the basic structure of the optimal debt contract is the same under complete and partial monitoring, the differences in the incentive compatibility constraints influence the rents to the different parties. In particular, the required contribution from the monitor will change in a way that will be shown shortly.

**Complete Monitoring.** As was the case in a merger without the possibility of monitoring, the constraint ensuring that working on both projects dominates shirking on both projects implies the constraint that working on both projects dominates working on only one project. The optimal contract for the monitor under complete monitoring is thus of the form \( \{R_M,0,0\} \) with the reward in case of two successes given by

\[
R_M = \frac{2c}{(p_H + p_L)\Delta p}
\]

Letting \( R_U \) denote the reward to the uninformed lenders in case of success, a sharing rule is given by

\[
R_F + R_U + R_M = R
\]

The expected return to the monitor is

\[
p_H^\beta R_M - 2c = \frac{2c}{(p_H + p_L)\Delta p} \frac{2c}{(p_H + p_L)\Delta p} - 2c = \frac{2cpld}{\Delta p} > 0
\]

As in the case with monitoring of a single project, the monitor receives a positive expected rent from monitoring. Let \( I_M(\beta) \) denote the monitor’s investment in the firm (per project). Assuming that the market for monitoring capital is competitive, the monitor must break even when signing the contract. He is thus required to invest at least

\[
I_M(\beta) \geq \frac{p_H(1-d)c}{\beta \Delta p}
\]

in the firm. As the outside opportunity to monitoring is to invest in the market for uninformed capital, it follows from familiar steps that

\[
I_M(\beta) \geq \frac{dplc}{\gamma \Delta p}
\]
Also, a little algebra shows that
\[ \beta = \frac{\gamma p_H^2}{p_L} > \gamma \]
so the firm will use the minimum possible amount of informed capital. Note that the return on monitoring capital with merger and complete monitoring \( \gamma p_H^2 / p_L^2 \) is larger than \( \gamma p_H / p_L \), which was the return on monitoring capital in the absence of merger possibilities.

Since the monitor is induced to monitor both projects, the agent’s opportunity cost of choosing \((G, G)\) is \(2s < 2S\). In turn, this implies that the pledgeable income is given by
\[ 2p_H R - p_H^2 R_F - p_H^2 R_M = 2p_H \left( R - \frac{(1 - d)(s + c)}{\Delta p} \right) \]
Since the uninformed investors must obtain at least \( p_H R_U = 2\gamma (I - K - I_M(\beta)) \), their individual rationality constraint is
\[ p_H \left( R - \frac{(1 - d)(s + c)}{\Delta p} \right) \geq \gamma (I - K - I_M(\beta)) \]
Consequently, the cut-off level for a merged firm to obtain finance with monitoring is given by
\[ D(\gamma, \beta) \equiv I - I_M(\beta) - \frac{p_H}{\gamma} \left( R - \frac{(1 - d)(s + c)}{\Delta p} \right) \]
Note that this amount is required per project. For \( K < D(\gamma, \beta) \), a merged firm with complete monitoring cannot obtain financing while it can for \( K \geq D(\gamma, \beta) \). Last, \( D(\gamma, \beta) \) is increasing in both arguments.

**Partial Monitoring.** The relevant incentive compatibility constraints ensuring that the firm chooses two good projects are now given by

\[ R_F \geq \frac{S}{p_H \Delta p} \]
\[ R_F \geq \frac{S + s}{(p_H + p_L) \Delta p} \]
The first constraint ensures that working on both projects dominates working on only the \( s \) project (and thus shirking on the \( S \) project), while the second constraint ensures that working on both projects dominates shirking on both. Unlike the case of complete monitoring, it is not generally true under partial monitoring that if one of the constraints is satisfied, then so is the other constraint. A little algebra shows that the first constraint implies the second constraint when
\[ \frac{p_H}{p_L} < \frac{S}{s} \equiv \sigma \]
where \( \sigma \) is a measure of the effectiveness of monitoring in reducing moral hazard in the firm’s choice of effort.

The two cases will be analyzed separately. In both cases, the monitor’s incentive compatibility constraint is given by
\[ R_M \geq \frac{c}{\Delta p} \]
as was the case under monitoring of a single firm without the possibility of merger. Consequently, in order to regulate the monitor’s rents, he is required to make a contribution towards the funding of the firm amounting to

\[ I_M(\beta) \geq \frac{p_H c}{\beta \Delta p} = \frac{p_L c}{\gamma \Delta p} \]

Since the contract for the monitor under merger and partial monitoring is essentially identical to that under monitoring in the absence of merger, it follows that the required return on the monitor’s contribution is given by

\[ \beta = \gamma \frac{p_H}{p_L} > \gamma \]

Note that this is lower than the required return on monitoring capital under complete monitoring.

**Case I.** First, I consider the case where \( \sigma \in [1, p_H/p_L] \). In this case, the relevant incentive compatibility constraint for the agent is given by

\[ R_F \geq \frac{S + s}{(p_H + p_L) \Delta p} \]

The merged firm’s pledgeable income is given by

\[ 2p_H R - p_H^2 R_F - p_H R_M = 2p_H \left( R - \frac{(1 - d)(S + s) + c}{2 \Delta p} \right) \]

Again letting \( K = (K' + K'')/2 \) denote the merged firm’s average per project net worth, the firm is funded only if

\[ 2p_H \left( R - \frac{(1 - d)(S + s) + c}{2 \Delta p} \right) \geq \gamma (2I - 2K - I_M(\beta)) \]

where \( p_H R_U = \gamma (2I - 2K - I_M(\beta)) \) is the minimum acceptable return to uninformed investors. Rearranging, the sought threshold on average net worth per project becomes

\[ E(\gamma, \beta) \equiv I - \frac{I_M(\beta)}{2} - \frac{p_H}{\gamma} \left( R - \frac{(1 - d)(S + s) + c}{2 \Delta p} \right) \]

For \( K < E(\gamma, \beta) \), a merged firm with partial monitoring cannot obtain financing while it can for \( K \geq E(\gamma, \beta) \). Last, \( E(\gamma, \beta) \) is increasing in both arguments.

**Case II.** Next, I consider the case where \( \sigma \in [p_H/p_L, \infty] \). In this case, the relevant incentive compatibility constraint for the agent is given by

\[ R_F \geq \frac{S}{p_H \Delta p} \]

Pledgeable income is then given by

\[ 2p_H R - p_H^2 R_F - p_H R_M = 2p_H \left( R - \frac{S + c}{2 \Delta p} \right) \]
and funding is secured if

\[ 2p_H \left( R - \frac{S + c}{2\Delta p} \right) \geq \gamma (2I - 2K - I_M(\beta)) \]

where again \( p_H R_U = \gamma (2I - 2K - I_M(\beta)) \) is the minimum acceptable return to uninformed investors. From this, the relevant threshold on average per project net worth is given by

\[ F(\gamma, \beta) \equiv I - \frac{I_M(\beta)}{2} - \frac{p_H}{\gamma} \left( R - \frac{S + c}{2\Delta p} \right) \]

For \( K < F(\gamma, \beta) \), a merged firm with partial monitoring cannot obtain financing while it can for \( K \geq F(\gamma, \beta) \). Last, \( F(\gamma, \beta) \) is increasing in both arguments.

3. Aggregate Demand

To sum up the analysis so far, firms are allowed to either (i) stay independent and seek direct finance, (ii) stay independent and seek indirect finance, (iii) merge and seek direct finance, (iv) merge and seek indirect finance of both projects and (v) merge and seek indirect finance of only one of the projects. Each type of financing has specific requirements on the firms’ net worth. For ease of reference, the different thresholds are represented in the following list:

- \( A(\gamma) \): no merger, no monitoring
- \( B(\gamma, \beta) \): no merger, monitoring
- \( C(\gamma) \): merger, no monitoring
- \( D(\gamma, \beta) \): merger, complete monitoring
- \( E(\gamma, \beta) \): merger, partial monitoring (case I)
- \( F(\gamma, \beta) \): merger, partial monitoring (case II)

In order to determine the aggregate demand for uninformed and monitoring capital, the ranking of the different thresholds must be established. Unfortunately, only a few of these rankings are unambiguous. Most depend on the parameter values. As shall be seen below, only one range of parameter values is relevant for the current analysis.

First, the cutoffs on net worth are ranked as follows, as functions of the monitoring cost:
Lemma 3. The rankings of the cutoffs for capital are given by

\[
\begin{align*}
A(\gamma) & \geq B(\gamma, \beta) \quad \text{for} \quad c \leq k_4 \\
A(\gamma) & \geq C(\gamma) \quad \text{for all} \quad c \\
A(\gamma) & \geq D(\gamma, \beta) \quad \text{for} \quad c \leq k_5 \\
A(\gamma) & \geq E(\gamma, \beta) \quad \text{for} \quad c \leq k_6 \\
A(\gamma) & \geq F(\gamma, \beta) \quad \text{for} \quad c \leq k_{AF} \\
B(\gamma, \beta) & \geq C(\gamma) \quad \text{for} \quad c \geq k_1 \\
B(\gamma, \beta) & \geq D(\gamma, \beta) \quad \text{for all} \quad c \\
B(\gamma, \beta) & \geq E(\gamma, \beta) \quad \text{for} \quad c \geq k_{BF} \\
C(\gamma) & \geq D(\gamma, \beta) \quad \text{for} \quad c \leq k_3 \\
C(\gamma) & \geq E(\gamma, \beta) \quad \text{for} \quad c \leq k_3 \\
C(\gamma) & \geq F(\gamma, \beta) \quad \text{for} \quad c \leq k_{CF} \\
D(\gamma, \beta) & \geq E(\gamma, \beta) \quad \text{for} \quad c \geq k_3 \\
D(\gamma, \beta) & \geq F(\gamma, \beta) \quad \text{for} \quad c \geq k_{DF}
\end{align*}
\]

where

\[
\begin{align*}
k_0 &= \frac{p_H ((1 - d) S - (1 + d) s)}{\Delta p} \\
k_1 &= \frac{p_H ((1 - d) S - s)}{\Delta p} \\
k_2 &= S - s \\
k_3 &= \frac{p_H (1 - d) (S - s)}{\Delta p} \\
k_4 &= \frac{p_H (S - s)}{\Delta p} \\
k_5 &= \frac{p_H (1 - d) (S - s) + p_L (1 - d) S}{\Delta p} \\
k_6 &= \frac{p_H (1 - d) (S - s) + 2 p_L (1 - d) S}{\Delta p} \\
k_{AF} &= \frac{p_H S}{\Delta p} \\
k_{BF} &= \frac{p_H (S - 2 s)}{\Delta p} \\
k_{CF} &= \frac{(1 - 2d) p_H S}{\Delta p} \\
k_{DF} &= \frac{(1 - d) (p_H + p_L) S - 2(1 - d) p_H s}{\Delta p}
\end{align*}
\]

Proof. The rankings follow from lengthy but straightforward algebra.

In turn, the ranking of cost cutoffs depends on the magnitude of the parameter \( \sigma \). Each case will be analyzed in turn.
3.1. Case I. First consider the case where \( \sigma \in [1, p_H/p_L] \). In this case, the relevant threshold for partial monitoring is \( E(\gamma, \beta) \) and the only relevant cutoffs for the monitoring cost is the sequence \( k_i, i = 0, ..., 6 \). Straightforward algebra shows that in Case I, the ranking is given as follows:

\[
k_6 \geq k_3 \geq k_4 \geq k_3 \geq k_2 \geq k_1 \geq k_0
\]

First, note that for \( c > k_4 \), \( A(\gamma) \leq B(\gamma, \beta) \) and thus monitoring is too costly to be socially useful. This is because for monitoring costs in this range, firms would not demand any informed capital even if the rate of return for monitors was so low that it just allowed them to break even. Thus the condition \( c \leq k_4 \) is imposed. Next, note that if \( c < k_2 \), the firms' pledgeable income (and thus their ability to raise uninformed capital) would be boosted by employing a monitor. In turn, this means that monitors would be able to monitor firms without having to inject any of their own funds into the monitored projects. Since the amount of informed capital is one of the important quantities with respect to which comparative statics will be performed, this possibility is excluded.\(^{14}\) It is therefore assumed that \( c \geq k_2 \). This leaves the interval \([k_2, k_3]\). But for \( c \geq k_3 \), \( C(\gamma) \leq E(\gamma, \beta) \), which means that it takes lower per project net worth to obtain funding as stand-alone projects than with partial monitoring of a merged firm. This means that there would be no demand for partial monitoring from merged firms, since monitoring reduces pledgeable income. This leaves the interval \([k_2, k_3]\). But for \( c \leq k_3 \), it is the case that \( E(\gamma, \beta) \leq D(\gamma, \beta) \), which means that it takes lower per project net worth to obtain funding for a merged firm with partial monitoring than it does with complete monitoring, in turn implying that no merged firms would demand complete monitoring. In conclusion, Case I is ruled out since for \( \sigma \in [1, p_H/p_L] \), not all markets can be simultaneously active.

3.2. Case II. Next, turn to case II, where \( \sigma \in [p_H/p_L, \infty] \). In this case, the relevant threshold for partial monitoring is \( F(\gamma, \beta) \). In contrast to case I where most of the critical thresholds coincide, case II breaks down to four further subcases.

Case II.a: \( \sigma \in [p_H/p_L, (p_H + p_L)/p_L] \). In this subcase, the cutoffs for monitoring cost \( c \) are ranked as follows:

\[
k_6 > k_{AF} > k_5 > k_4 > k_{DF} > k_3 > k_{CF} > k_2 > k_1 > k_{BF} > k_0
\]

The same arguments as those for case I above reduces the relevant range to \( c \in [k_2, k_3] \).\(^{15}\) But note that for \( c \geq k_{CF} \) it is the case that \( F(\gamma, \beta) \geq C(\gamma) \) which means that it is easier for a merged firm to get funded without monitoring than with partial monitoring. Thus, there would be no demand for partial monitoring from merged firms. This leaves the interval \( c \in [k_2, k_{CF}] \) as the only viable parameter range for this case.

Case II.b: \( \sigma \in [(p_H + p_L)/p_L, (p_H + 2p_L)/p_L] \). In this subcase, the cutoffs for monitoring cost \( c \) are ranked as follows:

\[
k_6 > k_{AF} > k_5 > k_4 > k_{DF} > k_3 > k_{BF} > k_1 > k_2 > k_{CF} > k_0
\]

---

\(^{14}\)For completeness, it should be noted that the equivalent conditions when comparing merger with complete monitoring to merger without monitoring and merger with partial monitoring to merger without monitoring, are that \( c > S - s \) and that \( c > (1 - d)(S - s) \), respectively. The former condition coincides with the condition \( c > k_3 \) while the latter is implied by it.

\(^{15}\)For completeness, it should be noted that the condition on \( c \) that ensures that pledgeable income under partial monitoring in Case II is not larger than pledgeable income under merger with no monitoring is that \( c \geq (1 - 2d)S \). But this is automatically satisfied in the parameter range under consideration, since then \( k_2 > (1 - 2d)S \).
The same arguments as those in cases I and II.a rule this case out since for this range of $\sigma$ it is the case that $k_{CF} \leq k_2$.

Case II.c: $\sigma \in [(p_H + 2p_L)/p_L, 2p_H + p_L]/p_L]$. In this case, the cutoffs are ranked as follows:

$$k_6 > k_{AF} > k_5 > k_4 > k_{DF} > k_{BF} > k_3 > k_1 > k_2 > k_0 > k_{CF}$$

The same arguments as those in cases I and II.a rule this case out since for this range of $\sigma$ it is the case that $k_{CF} \leq k_2$.

Case II.d: $\sigma \in [(2p_H + p_L)/p_L, \infty]$. In this case, the cutoffs are ranked as follows:

$$k_6 > k_{AF} > k_5 > k_4 > k_{DF} > k_{BF} > k_3 > k_1 > k_0 > k_2 > k_{CF}$$

The same arguments as those in cases I and II.a rule this case out since for this range of $\sigma$ it is the case that $k_{CF} \leq k_2$.

3.3. Parameterization. In conclusion, there is only one set of parameter ranges under which all markets are simultaneously active, namely that given by case II.a. In what follows, the following assumptions will made:

A1 $\sigma \in \left[ \frac{p_H}{p_L}, \frac{p_H + p_L}{p_L} \right]$

A2 $c \in \left[ S - s, \frac{p_H S}{p_H + p_L} \right]$

A3 $\Delta p(p_H R - \gamma I) \leq p_H (1 - d)(s + c) - dp_L c$

A4 $\Delta p(p_H R + \gamma L - \gamma I) \geq p_H S$

A5 $\frac{\beta}{\gamma} \geq \left( \frac{p_H}{p_L} \right)^2$

A6 $p_H R - \gamma I > 0 > p_L R - \gamma I + S$

A7 $c < \min\{\zeta(\gamma, \beta), \xi(\gamma, \beta)\}$, where

$$\zeta(\gamma, \beta) = \left( \frac{\beta}{\beta - 2\gamma} \right) \frac{2p_H^2 + p_H p_L - p_L^2}{2p_H^2 + p_H p_L + p_L^2} S - 2p_H^2 s$$

$$\xi(\gamma, \beta) = \left( \frac{\beta}{\beta - 2\gamma} \right) \frac{2p_H^2 + p_H p_L - p_L^2}{2p_H^2 + p_H p_L + p_L^2} S - 2p_H^2 s - 2\gamma I \left( p_H^2 - p_L^2 \right)$$

Assumptions A1-A2 correspond to the parameter range of case II.a discussed above. Assumptions A3-A4 ensure that no firm can obtain financing without supplying own funds and that the richest firm can obtain funding without monitoring. Assumption A5 ensures that the minimum return on monitoring capital makes informed investors willing to provide complete monitoring of merged firms. Assumption 6 ensures that a project is only viable if high effort is exerted. Last, assumption A7 implies that the two kinds of capital are gross substitutes.17

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16 Assumptions A3 and A4 are simply the inequalities $D(\gamma, \beta) \geq 0$ and $L \geq A(\gamma)$, respectively.

17 For completeness, it should be noted that $\zeta(\gamma, \beta) > \xi(\gamma, \beta)$ for $\beta > 2\gamma$ and $\zeta(\gamma, \beta) < \xi(\gamma, \beta)$ for $\beta < 2\gamma$. 

Under assumptions A1-A4, the ranking of the thresholds on net worth are given by

\[ I > L \geq A(\gamma) \geq B(\gamma, \beta) \geq C(\gamma) \geq F(\gamma, \beta) \geq D(\gamma, \beta) \geq 0 \]

Now the aggregate demands can be established. The demand for monitoring capital is given by

\[ D_M(\gamma, \beta) = [F(\gamma, \beta) - D(\gamma, \beta)] \frac{p_H(1 - d)c}{\beta \Delta p} + [C(\gamma) - F(\gamma, \beta)] \frac{p_H c}{\beta \Delta p} \tag{3} \]

Next, the demand for uninformed capital is given by

\[
\begin{align*}
D_U(\gamma, \beta) &= \int_{C(\gamma)}^{L} (I - \alpha K) dK + \int_{F(\gamma, \beta)}^{C(\gamma)} \left( I - \alpha K - \frac{p_H c}{\beta \Delta p} \right) dK \\
&\quad + \int_{D(\gamma, \beta)}^{F(\gamma, \beta)} \left( I - \alpha K - \frac{p_H(1 - d)c}{\beta \Delta p} \right) dK \\
&= I (L - D(\gamma, \beta)) - \frac{\alpha L^2}{2} - \frac{p_H c}{\beta \Delta p} [C(\gamma) - dF(\gamma, \beta) - (1 - d)D(\gamma, \beta)]
\end{align*}
\]

In this demand function, each firm’s assets \( K \) have been scaled by a factor \( \alpha \geq 1 \) so the effects of an increase in collateral can be studied. Last, note that firms with \( K \in [B(\gamma, \beta), A(\gamma)] \) could choose to remain independent and raise funds with the aid of an intermediary. Since intermediation is costly and no cost to mergers has been assumed, it is natural to suppose that these firms will indeed choose to diversity.\(^\ddagger\)

The slopes of the demand equations are as follows:

\[
\begin{align*}
\frac{\partial}{\partial \gamma} D_M(\gamma, \beta) &= \frac{cp_H^2 \left[ c (2p_H^2 + phpL + p_L^2) - S (2p_H^2 + phpL - p_L^2) + 2p_H^2 s \right]}{2(p_H + p_L)^2 \beta^2 \gamma^2 \Delta p^2} \tag{5} \\
\frac{\partial}{\partial \beta} D_M(\gamma, \beta) &= \frac{cp_H^2 \left[ \frac{(\beta - 2\gamma)}{\beta} c (2p_H^2 + phpL + p_L^2) - S (2p_H^2 + phpL - p_L^2) + 2p_H^2 s \right]}{2(p_H - p_L)^2 \beta^2 \gamma^2 \Delta p^2} \tag{6} \\
\frac{\partial}{\partial \gamma} D_U(\gamma, \beta) &= -\frac{cp_H^2 \left[ c (2p_H^2 + phpL + p_L^2) - S (2p_H^2 + phpL - p_L^2) + 2p_H^2 s \right]}{2(p_H + p_L)^2 \beta^2 \gamma^2 \Delta p^2} - \frac{p_H \left[ R - \frac{(1 - d)(c + s)}{\Delta p} \right]}{\gamma^2} \tag{7} \\
\frac{\partial}{\partial \beta} D_U(\gamma, \beta) &= -\frac{cp_H^2 \left[ \frac{(\beta - 2\gamma)}{\beta} c (2p_H^2 + phpL + p_L^2) - S (2p_H^2 + phpL - p_L^2) + 2p_H^2 s \right]}{2(p_H + p_L)^2 \beta^2 \gamma^2 \Delta p^2} \tag{8}
\end{align*}
\]

Under assumptions A1-A2, (5) is positive while (7) is negative. Under assumption A7, (6) is negative while (8) is positive.

\(^\ddagger\)Note that firms are assumed to only merge within groups. Thus, a rich firm who can access the capital markets directly will not be allowed to merge with a poor firm. I consider this setting to ensure that there will be positive demand for financial intermediation. Furthermore, such settings in which cash-rich firms fund poor firms has been considered elsewhere in the literature, e.g. by Fluck and Lynch (1999).
To sum up, the chosen parameterization of the model ensures that all markets for monitoring are active. Furthermore, it ensures that the demand functions for capital are downward sloping in own price and increasing in the price of the other type of capital.\footnote{Note for later use that \( \lim_{\gamma \to 0} D_U(\gamma, \beta) = \infty \) while \( \lim_{\gamma \to \infty} D_U(\gamma, \beta) = \lambda < 0 \) where \( \lambda \) is a finite constant. That \( \lambda \) is a finite constant follows directly from taking the limit. To see that it must be negative, note that for \( \gamma > \mu_H R/I \) the net value of projects is negative and so demand cannot be positive.} This is consistent with the finding of Kashyap et al. (1993) that commercial papers and bank loans are imperfect substitutes.

4. Equilibrium and Comparative Statics

Two scenarios will be analyzed, each differing in its assumption about the market for uninformed capital. In the first, it is assumed that there is an infinite supply at rate \( \gamma \) while in the second, uninformed capital is supplied according to an upward-sloping supply function \( S(\gamma) \). The following definitions and results shall prove useful in what follows:

\[ A(\gamma, \beta) \equiv L - D(\gamma, \beta), \quad P(\gamma, \beta) \equiv C(\gamma) - F(\gamma, \beta), \quad C(\gamma, \beta) \equiv F(\gamma, \beta) - D(\gamma, \beta), \quad M(\gamma, \beta) \equiv A(\gamma) - D(\gamma, \beta) \]

Furthermore, under assumptions A1-A4,

\[ \frac{\partial A(\gamma, \beta)}{\partial \beta} < 0, \quad \frac{\partial P(\gamma, \beta)}{\partial \beta} < 0, \quad \frac{\partial C(\gamma, \beta)}{\partial \beta} < 0, \quad \frac{\partial M(\gamma, \beta)}{\partial \beta} < 0 \]

\[ \frac{\partial A(\gamma, \beta)}{\partial \gamma} < 0, \quad \frac{\partial P(\gamma, \beta)}{\partial \gamma} > 0, \quad \frac{\partial C(\gamma, \beta)}{\partial \gamma} < 0, \quad \frac{\partial M(\gamma, \beta)}{\partial \gamma} < 0 \]

4.1. Infinitely Elastic Supply. In this scenario, equilibrium is described entirely by equilibrium in the market for monitoring capital, which is described by the equation

\[ e_M(\gamma, \beta) \equiv D_M(\gamma, \beta) - K_M = 0 \]

Under the maintained assumptions, demand for monitoring capital is downward-sloping. Since \( \lim_{\beta \to 0} D_M(\gamma, \beta) = \infty \) and \( \lim_{\gamma \to \infty} D_M(\gamma, \beta) = 0 \), continuity and monotonicity imply both the existence and uniqueness of equilibrium in the market for informed capital.

From (11), it is immediately clear that an increase in the amount of monitoring capital decreases the equilibrium rate of return \( \beta \). In turn, inequalities (9) imply the following:

**Proposition 5.** In a credit crunch, (i) the rate of return on informed capital \( \beta \) increases, (ii) the set of merged firms decreases, (iii) the set of firms with partial monitoring decreases, (iv) the set of firms with complete monitoring decreases and (v) the set of all active firms decreases.

As the proposition shows, a credit crunch decreases the set of active firms by squeezing poorly capitalized firms out of the market, much as in Holmström and Tirole (1997). This decrease of active firms is brought about in part by a decrease in the set of merged firms, both partially and completely monitored ones. Next, consider the effects of a change in the exogenous interest rate \( \gamma \). Because (5) is positive, the following can be established:
Proposition 6. With an increase in the rate of return $\gamma$, (i) the rate of return on informed capital $\beta$ increases, (ii) the set of merged firms decreases, (iii) the set of firms with partial monitoring decreases, (iv) the set of firms with complete monitoring decreases and (v) the set of all active firms decreases.

Thus the effects of an increase in the exogenous interest rate, qualitatively mirror those of a credit crunch.

These observations accord well with the stylized facts that merger activity is to be expected to increase in times of economic expansion. During good times, interest rates decrease as do the non-interest hurdles to raising finance. In turn, this allows poorly capitalized firms raise funds for projects and thereby become economically active. The model thus predicts that in prosperous times of falling interest rates and eased access to funding, both industrial output, business incorporations and merger activity should rise, in accordance with the empirical regularities reviewed in the introduction.

4.2. Elastic Supply. When the supply of uninformed capital is not infinitely elastic, an equilibrium condition for the market for uninformed capital must be added. Equilibrium in the market for uninformed capital is described by the equation

$$e_U(\gamma, \beta) \equiv D_U(\gamma, \beta) - S(\gamma) = 0$$

Equilibrium in the two markets obtains when (11) and (12) are satisfied.

Next, the two equilibrium conditions will be studied in $(\gamma, \beta)$-space. By the implicit function theorem and the signs of (5)-(8), the slopes of the equilibrium conditions are given by

$$\frac{d\beta}{d\gamma}\bigg|_{e_M(\gamma, \beta)=0} = -\frac{\frac{\partial}{\partial \gamma}D_M(\gamma, \beta)}{\frac{\partial}{\partial \beta}D_M(\gamma, \beta)} > 0$$

$$\frac{d\beta}{d\gamma}\bigg|_{e_U(\gamma, \beta)=0} = -\frac{\frac{\partial}{\partial \gamma}D_U(\gamma, \beta) - S'(\gamma)}{\frac{\partial}{\partial \beta}D_U(\gamma, \beta)} > 0$$

In other words, uninformed and informed capital are gross substitutes. Since both equilibrium conditions (11) and (12) are upward-sloping in $(\gamma, \beta)$-space, there are potentially multiple equilibria. This means that comparative statics predictions may be reversed when moving from one equilibrium to another. Before considering the comparative statics of the model, I will next discuss these issues further.

Existence, Uniqueness and Stability of Equilibrium. Equilibrium uniqueness cannot be expected in the current model. A sufficient condition for a unique stable equilibrium to exist will be given below. For there to be at most one equilibrium, it must be shown that there is at most one pair $(\gamma, \beta)$ such that (11)-(12) hold. This is the case if one of the functions is everywhere steeper than the other in $(\gamma, \beta)$-space. The condition for uniqueness to obtain and a necessary condition for that equilibrium to be stable is that

$$\frac{d\beta}{d\gamma}\bigg|_{e_M(\gamma, \beta)=0} < \frac{d\beta}{d\gamma}\bigg|_{e_U(\gamma, \beta)=0}$$
This condition is equivalent to

$$S'(\gamma) > \frac{\partial^2 D_U(\gamma, \beta)\frac{\partial}{\partial \gamma} D_M(\gamma, \beta) - \frac{\partial}{\partial \gamma} D_M(\gamma, \beta)\frac{\partial}{\partial \beta} D_U(\gamma, \beta)}{\partial^2 D_M(\gamma, \beta)}$$

The right-hand side of inequality (15) is however of unknown sign and thus uniqueness cannot be ensured. Therefore, the analysis will henceforth be restricted to the (possibly non-singleton) set of stable equilibria, i.e. those intersections of (11)-(12) where (12) intersects (11) from below in ($\gamma$, $\beta$)-space.

Next, define the relative price $\rho \equiv \beta / \gamma$ and let $\gamma = 1$ be the numeraire. The aggregate excess demand function is then given by

$$e(\rho) \equiv D_U(1, \rho) + D_M(1, \rho) - S(1) - K_M$$

In order to ensure existence of a stable equilibrium satisfying condition A5, the following sufficient condition is imposed:

$$\text{A8} \lim_{\rho \to (p_H/p_L)^2} [D_U(1, \rho) + D_M(1, \rho)] > K_M + S(1) > \lim_{\rho \to \infty} D_U(1, \rho)$$

The second inequality ensures that an equilibrium set of prices $(\gamma, \beta)$ exists and the first inequality ensures that this equilibrium set of prices makes the monitors willing to provide complete monitoring of merged firms.\(^{20}\)

**Comparative Statics.** Next, I will determine how the equilibrium conditions shift in response to changes in collateral, savings and aggregate monitoring capital.

In a credit crunch, the aggregate amount of monitoring capital $K_M$ decreases. This in effect increases the required rate of return on informed capital $\beta$, thus shifting the function (11) leftward in ($\gamma$, $\beta$)-space. In any stable equilibrium, this shift increases both $\gamma$ and $\beta$.

In a savings squeeze, the supply function $S(\gamma)$ shifts rightward for any given value of $\beta$. In consequence, equilibrium on the market for uninformed capital is now at a higher rate of return $\gamma$. This means that the function (12) in ($\gamma$, $\beta$)-space shifts rightward. In any stable equilibrium, this shift increases both $\gamma$ and $\beta$.

In a collateral squeeze, the firm’s asset holdings are eroded. This corresponds to a decrease in $\alpha$ in the current setting.\(^{21}\) Such a decrease in collateral shifts the demand function (4) rightward in ($\gamma$, $\beta$)-space, thereby increasing the return on uninformed capital $\gamma$ for any given level of $\beta$. This has the effect of shifting the function (12) rightward. There is no effect of changes in $\alpha$ on the demand for informed capital (3). In any stable equilibrium, this shift increases both $\gamma$ and $\beta$.

Since the two rates of return $\gamma$ and $\beta$ move in the same direction in equilibrium as the equilibrium conditions (11)-(12) shift, it is possible to characterize the effects of such shifts on the magnitudes of the different sets of firms under consideration. In equilibrium, $\gamma$ and $\beta$ must simultaneously satisfy (11) and (12). It then follows from the implicit function theorem that $\beta$ can be viewed as a function of $\gamma$. The next step is to determine the overall effect on the different sets of agents of changes in $\gamma$, taking into account both the direct effect through

\(^{20}\)Although $\lim_{\rho \to 0} D_M(1, \rho) = \infty$ and $\lim_{\rho \to \infty} D_M(1, \rho) = 0$, the fact that $D_U(\gamma, \beta)$ increases in $\beta$ makes condition A8 necessary for equilibrium existence. For completeness, it should also be mentioned that the interval defined by the inequalities in assumption A8 has positive measure.

\(^{21}\)Note that the decrease in assets is proportional to each firm’s initial assets.
γ and the indirect effect through β. Since γ and β move together, the overall effect can be determined whenever these two effects are of the same sign. From inequities (9)-(10), the following can be established:

**Proposition 7.** In any stable equilibrium, a credit crunch, a savings squeeze or a collateral squeeze (i) increases the rate of return on uninformed capital γ and the rate of return on informed capital β, (ii) decreases the set of merged firms, (iii) decreases the set of firms with complete monitoring and (iv) decreases the set of all active firms. The effects on the set of firms with partial monitoring are ambiguous.

This proposition shows that, with the exception of the set of partially monitored firms, the comparative statics results obtained under the assumption of infinitely elastic supply of uninformed capital are robust to the extension to a setting with endogenous supply of uninformed capital. In particular, it establishes that business incorporations, industrial output and merger activity should all be expected to rise under conditions of low interest rates and easy access to finance. In contrast to the setting with infinitely elastic supply of uninformed capital, this version of the model also allows for a study of the effects of collateral values on merger activity. As noted by Shleifer and Vishny (1992), the fact that collateral values are procyclical may suggest a link between collateral values and aggregate activity. Such a link is present in the current model, as increases in the value of collateral lets more firms at the lower end of the capital distribution become economically active, firms who must merge in order to raise external funds.

One interesting possibility of the current model is that the composition of external finance, i.e. the relative magnitudes of informed and uninformed capital, may change in predictable directions as economic conditions change. Kashyap et al. (1993) have studied such changes in the composition of external financing for a sample of firms (not necessarily diversified firms) and found that tightening of monetary policy indeed has important effects. They find that when monetary policy tightens, issuance of commercial paper rises at the expense of bank loans. This suggests that the two types of financing are imperfect substitutes as is the case in the present model. In future work, it would be interesting to conduct a study of diversified firms in the spirit of Kashyap et al. (1993) and to relate the findings with the predictions of the present theory.

A couple of thoughts on the nature of mergers considered in the current work are in order. Prima facie, the developed theory is one of pure conglomerate mergers, as the projects which make up a merged entity are assumed independent. Note though, that independence is not necessary for diversification purposes. Specifically, if projects are less than perfectly correlated, some diversification possibilities will be present (see Diamond, 1984). In practice, not even the earnings of firms that are horizontally related are perfectly correlated. E.g., horizontally related firms that cover different geographical markets, who employ different input suppliers or who make use of different technologies do not have identical cash flows. This means that in practice, most if not all mergers will reduce the combined entity’s exposure to risk, regardless of the nature of the merger (be it horizontal, vertical or purely conglomerate).

5. **Diversification and the Value of Firms**

In this section, the value of firms undertaking diversifying mergers will be analyzed and the findings related to the diversification debate outlined in the introduction.
Let the net value of a firm, as a function of initial asset holdings $K$, be defined as $V(K) = E[R_F] - \gamma K$, where expectations are determined according to the relevant probabilities of success computed in the different subsections above. A little algebra shows that this value is given as follows:

$$V(K) = \begin{cases} 
0 & \text{for } K \in [0, D(\gamma, \beta)] \\
p_H R - \gamma I - c & \text{for } K \in [D(\gamma, \beta), F(\gamma, \beta)] \\
p_H R - \gamma I - c/2 & \text{for } K \in [F(\gamma, \beta), C(\gamma)] \\
p_H R - \gamma I & \text{for } K \in [C(\gamma), L]
\end{cases}$$

The average value of diversified firms is thus given by

$$V^D(K) = \frac{[F(\gamma, \beta) - D(\gamma, \beta)](p_H R - \gamma I - c)}{A(\gamma) - D(\gamma, \beta)} + \frac{[C(\gamma) - F(\gamma, \beta)](p_H R - \gamma I - c/2)}{A(\gamma) - D(\gamma, \beta)} + \frac{[A(\gamma) - C(\gamma)](p_H R - \gamma I)}{A(\gamma) - D(\gamma, \beta)}$$

In turn, the average value of independent (non-diversified) firms is given by

$$V^I(K) = p_H R - \gamma I$$

Clearly, the average value of diversified firms is lower than that of non-diversified firms, i.e. $V^D(K) < V^I(K)$. But note that this “discount” of diversified firms is not in fact an effect of the diversification per se, but rather a reflection of the fact that those firms who choose to diversify tend to be low value firms.

In assessing the value of diversification, an appropriate benchmark must be chosen. If a diversified firm is compared to a non-diversified firm, the diversification may seem to reduce value. But non-diversified firms are not the appropriate benchmark in the current setup. Rather, one must ask the question of what the value of a diversified firm would be in the absence of diversification. In the present model, the appropriate benchmark is firms with assets in the interval $[0, D(\gamma, \beta)]$, namely the firms that cannot obtain funding under any circumstances. Compared to this benchmark, diversification is clearly value enhancing, since $V^D(K) > 0$ and not getting funding is the alternative to not diversifying.

How does this finding relate to the diversification debate? In the current setting, the decision to diversify through mergers is endogenous, which means that I can characterize the firms that will choose such diversification in equilibrium. These are exactly the firms that must rely on costly financial intermediation which decreases firm value. The considered environment is rich enough to be able to differentiate between firms along more than one dimension, which enables me to drive a wedge between the diversification and the existence of a diversification discount (although of course, there is no wedge between the decision to diversify and the discount in the value of diversified firms). Existing literature cannot make such distinctions, as the only feature that differentiates between diversifying and non-diversifying firms in existing models is diversification itself.
6. Discussion

The present paper considered an equilibrium model in which cash-poor firms are credit rationed due to moral hazard problems. Two ways of alleviating the incentive problem were considered, namely diversifying mergers (which boost debt capacity) and the employment of a financial intermediary (which is costly but reduces the incentive problem). It was shown that in equilibrium, the poorest firms will opt for a combination of mergers and intermediation. This has several implications. First, diversifying firms are on average worth less than their non-diversifying counterparts. Second, when the cost of financial intermediation is low, as should be expected at the height of the business cycle, more firms become active and many of these will be diversified firms. Thus this model predicts that diversifying merger activity should be procyclical, a time series feature of M&A activity previously established in the empirical literature. Furthermore, the analysis showed that high merger activity should coincide with high economic activity, with a large number of business incorporation and with easy and cheap access to credit. Again, empirical findings are consistent with such predictions.

While diversification was emphasized in the current work as the main motivating force behind mergers, there is no denying that other reasons for mergers may be present, such as synergies, economies of scale, replacement of inefficient management etc. What this work shows is that even if one abstracts away these other (perfectly legitimate) motives for mergers, diversification alone does seem to explain empirically robust stylized facts of merger activity.
References


